Problems 3c)

$T : \mathbb{P}_n \to \mathbb{P}_n$

$Tf = f'$

$1 \to 0 \quad (0 \ldots 0)^T$

$z \to 1 \quad (1 0 \ldots 0)^T$

$z^2 \to 2z \quad (0 2 0 \ldots 0)^T$

$\vdots$

$z^k \to k z^{k-1} \quad (0 \ldots 0 \overset{k-1}{k} 0 \ldots 0)$

$\vdots$

$z^n = n z^{n-1}$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 2 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 3 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & 0 & 4 & & \vdots \\ \vdots & \vdots & \vdots & \vdots & 0 & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & n \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix} = A$$

Square matrix $(n+1) \times (n+1)$

3d.)

$Tf = 2f + 3f' - 4f''$

f''

$1 \to 0$

$z \to 0$

$z^2 \to 2$

$z^3 \to 2 \cdot 3 z$

$z^4 \to 4 \cdot 3 z^2$

$z^5 \to 5 \cdot 4 z^3$

$\vdots$

$z^n \to n(n-1) z^{n-2}$

$$\begin{pmatrix} 0 & 0 & 2 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 2 \cdot 3 & 0 & 0 & \cdots & 0 \\ \vdots & & 0 & 0 & 4 \cdot 3 & 0 & \cdots & 0 \\ \vdots & & & & 0 & 5 \cdot 4 & \cdots & 0 \\ \vdots & & & & & & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & & n \cdot (n-1) \\ & & & & & & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix} = B$$

$Tf = 2f$

$2I$

$$\begin{pmatrix} 2 & 0 & \cdots & 0 \\ 0 & 2 & & \vdots \\ \vdots & & \ddots & \\ \vdots & & 2 & 0 \\ 0 & \cdots & 0 & 2 \end{pmatrix}$$

$$2I + 3A - 4B = \begin{pmatrix} 2 & 3 & -8 & 0 & \cdots & & 0 & 0 \\ 0 & 2 & 6 & -24 & \cdots & & 0 & 0 \\ 0 & 0 & 2 & 9 & -40 & \cdots & & \\ \vdots & & & \ddots & \ddots & \ddots & & -4 \cdot n \cdot (n-1) \\ 0 & 0 & 0 & 0 & 0 & \cdots & 2 & 3n \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 2 \end{pmatrix}$$

Applications to Computer Graphics (7.1)

2-D Graphical object is collection of points on x-y plane

Translation:

$$\vec{v} \to \vec{v} + \vec{a}$$

Rotation:

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

Scaling

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

Reflection:

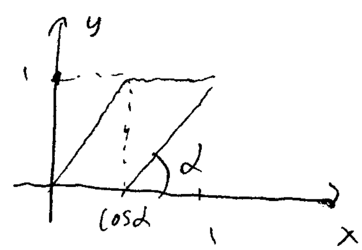
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Projection:

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Shear:

$$\begin{pmatrix} 1 & \cos\alpha \\ 0 & 1 \end{pmatrix}$$



3-D Graphics ($\mathbb{R}^3$)

Translation:

Rotation:

$$\begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

about z-axis

Scaling:

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

Reflection:

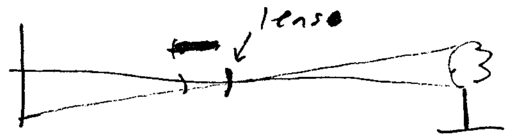
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

For 3-D image on 2-D plane:

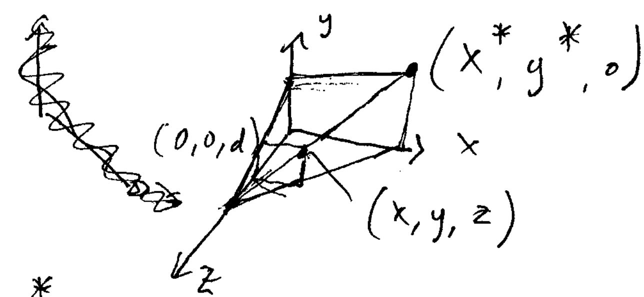
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 \end{pmatrix}$ Projection on xy plane

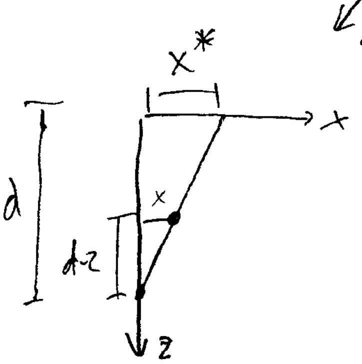
- picture doesn't account for perspective

Think of camera



Perspective projection





similar triangles:

$$\frac{x^*}{x} = \frac{d}{d-z}$$

$$x^* = \frac{xd}{d-z} = \frac{x}{1-\frac{z}{d}} \qquad y^* = \qquad = \frac{y}{1-\frac{z}{d}}$$

not linear

(x, y, z) $\quad (x_1 \quad x_2 \quad x_3 \quad x_4)$

$$\left(\frac{x_1}{x_4}, \frac{x_2}{x_4}, \frac{x_3}{x_4}, 1\right)$$

$$\begin{pmatrix} x \\ y \\ 0 \\ 1-\frac{z}{d} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{d} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\vec{v} \mapsto \vec{v} + \vec{a}$$

$$\begin{pmatrix} x+a_1 \\ y+a_2 \\ z+a_3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & a_2 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$