

MA 54 classnotes

2/25/03

Change of BasisLet $V = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots, \vec{v}_n\}$ be some basis B $\forall \vec{v} \in V$ admits unique representation

$$\vec{v} = \sum_{k=1}^n x_k \vec{v}_k$$

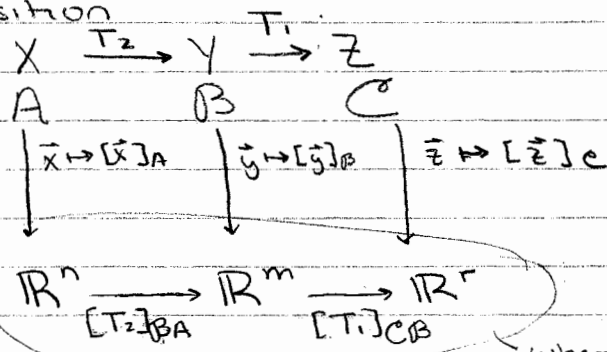
$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} =: [\vec{v}]_B \text{ - called coordinate representation of } \vec{v} \text{ in the basis } B \text{ or relative to the basis } B$$

Coord representation $\vec{v} \mapsto [\vec{v}]_B$ is an isomorphism between V and $\mathbb{R}^n$ Suppose 2 spaces X and Y and linear transformation $T: X \rightarrow Y$ $A = \{\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n\} \rightarrow$ basis in X $B = \{\vec{b}_1, \vec{b}_2, \dots, \vec{b}_n\} \rightarrow$ basis in Y take $\vec{x} \in X$ and let $\vec{y} = T\vec{x}$ How $[\vec{y}]_B, [\vec{x}]_A$ are related?

$$[\vec{y}]_B, [\vec{x}]_A$$

 $\leftarrow$ a unique

$$\exists ! \text{ matrix } [T]_{BA} \text{ s.t. } [\vec{y}]_B = [T]_{BA} [\vec{x}]_A$$

How to find $[T]_{BA}$?Take $[T\vec{a}_1]_B, [T\vec{a}_2]_B, \dots, [T\vec{a}_n]_B$ and join them in a matrix.Matrix of compositionSuppose we have spaces
and basesLet $T = T_1 T_2: X \rightarrow Z$

$$[T]_{CA} = [T_1]_{CB} [T_2]_{BA}$$

where computer does operation:

2/25/04

Change of coordinate matrix

X $A = \{\vec{a}_1, \dots, \vec{a}_n\}$ $B = \{\vec{b}_1, \dots, \vec{b}_n\}$
 $[I]_{BA} \leftarrow$ takes coordinates in A basis and gives out coordinates
 in B basis

$$[\vec{x}]_B = [I]_{BA} [\vec{x}]_A$$

↑ change of coordinate matrix

Remark:

$$[I]_{AB} = [I]_{BA}^{-1}$$

Ex $T = \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix}$ $B = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$

Want to find $[T]_{BB}$ $\Delta = \{\vec{e}_1, \vec{e}_2\}$ - standard basis

know that:

$$[T]_{\Delta\Delta} = \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix}$$

$$[T]_{BB} = [I]_{B\Delta} [T]_{\Delta\Delta} [I]_{\Delta B}$$

$$[I]_{\Delta B} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \quad [I]_{B\Delta} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}^{-1} = \frac{1}{3} \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$$

$$[T]_{BB} = \frac{1}{3} \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

if $[I]_{\Delta B} = Q$ then $[I]_{B\Delta} = Q^{-1}$

Ex

$\mathbb{R}_1$ $A = \{1, 1+t\}$ $B = \{1+2, 1-2t\}$ $\Delta = \{1, t\}$

Find: $[I]_{BA} = [I]_{B\Delta} [I]_{\Delta A}$

$$[I]_{\Delta A} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$[I]_{B\Delta} = [I]_{\Delta B}^{-1} \quad [I]_{\Delta B} = \begin{pmatrix} 1 & 1 \\ 2 & -2 \end{pmatrix}$$

$$[I]_{B\Delta} = \begin{pmatrix} 1 & 1 \\ 2 & -2 \end{pmatrix}^{-1}$$

$$[I]_{BA} = \begin{pmatrix} 1 & 1 \\ 2 & -2 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$