2/4/04

Composition of linear transformations & matrix multiplication

$A \cdot B$

Let $\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_r$ be columns of B.

So, $B = [\vec{b}_1, \vec{b}_2, \ldots \vec{b}_r]$

Then $A \cdot B = [\underbrace{A \cdot \vec{b}_1}_{1^{st}}, \underbrace{A \cdot \vec{b}_2}_{2^{nd}}, A \cdot \vec{b}_3, \ldots, A\vec{b}_r]$

column

Row by column rule.

$(A \cdot B)_{jk} = (\text{row } \# j \text{ of } A) \cdot (\text{col } \# k \text{ of } B)$

Example.

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2+2+0 & 1+6+1 \\ 0+1+0 & 0+3+2 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \text{ - not defined}$$

$A \cdot B$ is defined if size of row of A = size of col. of B.

$A_{m\times n} \cdot B_{n\times r} = AB_{m\times r}$

$T_1 : \mathbb{R}^n \to \mathbb{R}^m$

$T_2 : \mathbb{R}^r \longrightarrow \mathbb{R}^n.$

Let $T = T_1 \circ T_2$

$T(\vec{x}) = T_1 \circ T_2(\vec{x}) = T_1\underbrace{(T_2\vec{x})}_{\in \mathbb{R}^n} \quad \vec{x} \in \mathbb{R}^r.$

$A = [T_1]\ m\times n.$
$B = [T_2]\ n\times r$
$[T_1 \circ T_2]\ m\times r \longrightarrow$ matrix T

<u>How do we get matrix T?</u>

Take standard basis $\vec{e}_1, \vec{e}_2, \ldots \vec{e}_r.$

Calculate $T\vec{e}_1, T\vec{e}_2 \ldots T\vec{e}_r$

$T(\vec{e}_K) = T_1(T_2(\vec{e}_K)) = T_1(B\vec{e}_K)$ col# k of B, $\vec{b}_k$

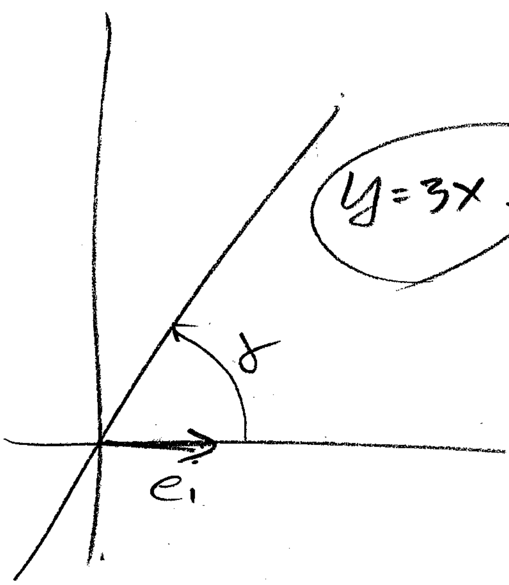
$= T_1(\vec{b}_K) = A \cdot \vec{b}_K.$

In matrix multiplication, usual rules of Algebra apply except that

$AB \neq B \cdot A$.

$$\left.\begin{aligned}&(AB)C = A(BC)\\&A(B+C) = AB + AC = AC + AB.\\&(B+C)\cdot A = BA + CA.\end{aligned}\right\}\ \text{true}$$

Problem



$$T_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$T = R_\gamma T_0 R_\gamma$$

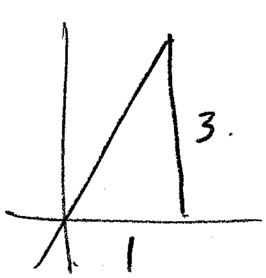
$$R_\gamma \ \vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} \cos\gamma \\ \sin\gamma \end{pmatrix}$$

$$\vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} -\sin\gamma \\ \cos\gamma \end{pmatrix}$$

$$R_\gamma = \begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix}$$

$$R_{-\gamma} = \begin{pmatrix} \cos(-\gamma) & -\sin(-\gamma) \\ \sin(-\gamma) & \cos(-\gamma) \end{pmatrix} = \begin{pmatrix} \cos(\gamma) & \sin(\gamma) \\ -\sin(\gamma) & \cos(\gamma) \end{pmatrix}$$

What is γ?



$$\cos\gamma = \frac{1}{\sqrt{1^2+3^2}} = \frac{1}{\sqrt{10}}.$$

$$\sin\gamma = \frac{3}{\sqrt{10}}$$

$$T = \begin{pmatrix} 1/\sqrt{10} & -3/\sqrt{10} \\ 3/\sqrt{10} & 1/\sqrt{10} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \cancel{1/\sqrt{10}} & 3/\sqrt{10} \\ -3/\sqrt{10} & 1/\sqrt{10} \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -3 & 1 \end{pmatrix}$$