John Shields
Math 54
Jan 28, 2004

## Vector Space

Two Operations:

① addition: vector + vector

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + y_n \end{pmatrix}$$

② multiplication: scalar • vector

$$\alpha \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} \alpha \cdot x_1 \\ \alpha \cdot x_2 \\ \vdots \\ \alpha \cdot x_n \end{pmatrix}$$

## Properties of Vectors

① $\vec{x} + \vec{y} = \vec{y} + \vec{x}$ ② $(\vec{x} + \vec{y}) + \vec{z} = \vec{x} + (\vec{y} + \vec{z})$

③ $\exists \vec{0}$ so that $\vec{0} + \vec{x} = \vec{x}$ ④ $\forall \vec{x} \exists (-\vec{x})$ so that $\vec{x} + (-\vec{x}) = \vec{0}$

⑤ $1 \cdot \vec{x} = \vec{x} \quad \forall \vec{x}$ ⑥ $(\alpha)\beta\vec{x} = (\beta)\alpha\vec{x}$

⑦ $(\alpha + \beta)\vec{x} = \alpha\vec{x} + \beta\vec{x}$ ⑧ $\alpha(\vec{x} + \vec{y}) = \alpha\vec{x} + \alpha\vec{y}$

Use rules much like those of algebra when dealing with vectors – EXCEPT that you cannot multiply vectors together

[redacted]

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \\ 5 \\ 6 \end{pmatrix} \Rightarrow$$ not defined, must add columns of the same size

$\uparrow R^3$ $\quad \uparrow R^4$

$-\vec{x} = (-1)\vec{x} \qquad \vec{0} = 0 \cdot \vec{x}$

## Polynomials

$\mathbb{P}_n$ polynomials of degree $\leq n$

$p(t) = a_0 + a_1 t + a_2 t^2 + \dots + a_n t^n$ $\quad p \sim \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \simeq \mathbb{R}^{n+1}$

## Matrices

$M_{m \times n}$

$\uparrow \quad \uparrow$

rows columns

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

If coordinates were put in one column,
$M_{m \times n} \sim \mathbb{R}^{m \cdot n}$

## Scalars

$\mathbb{R} = \mathbb{R}^1$ $\quad (x)$ $\quad$ A scalar is a column of size 1

$\{\vec{0}\}$ $\quad \emptyset$ – not a vector space

## Other

x-y plane $= \mathbb{R}^2$