Feb 27's HW problem #8

$n\times n$ Columns of A are LI $\Rightarrow$ rows of A^3 are LI

Pf

Columns of A are LI

(*) $\Rightarrow$ pivots in every column

$\Rightarrow n$ pivots

$\Rightarrow$ pivots in every row $(n\times n)$

with (*) $\Rightarrow A$ is invertible

$\Rightarrow A^3$ invertible $\left((A^3)^{-1} = (A^{-1})^3\right)$

$\Rightarrow A^{3^T}$ invertible $(B^T)^{-1} = (B^{-1})^T$

$\Rightarrow$ pivot in every column of A^T

$\Rightarrow$ columns of $(A^3)^T$ (i.e. rows of A^3) are LI

HW Due today problem 6

$$\begin{vmatrix} A & B \\ 0 & C \end{vmatrix} \rightarrow C_1 \begin{vmatrix} A_{UT} & B \\ 0 & C \end{vmatrix} \longrightarrow C_1 C_2 \begin{vmatrix} A_{UT} & * \\ 0 & C_{UT} \end{vmatrix}$$

Determinants

Prop $\det A = 0$ IFF A is not invertible

$\Leftarrow$ was before

$\Rightarrow$ If $\det A = 0$ then in the triangular form there is a 0 on diagonal

$\Rightarrow$ not every row (column) has a pivot

$\Rightarrow A$ is not invertible

Determinant of Product and Transpose

L_1 Let A be $n\times n$, E – elementary matrix, then $\det(AE) = \det A \det E$

Pf

Right multiplication by E performs column operation.

The effect of column operation is mult by $\det E$

L Any invert B can be represented as product of elementary matrices.

Pf B is row equivalent to its reduced echelon form.

B is invertible. $\Rightarrow B \sim I$

$I = E_N \dots E_2 E_1 B \Rightarrow B = E_1^{-1} E_2^{-1} \dots E_N^{-1}$

Thm $\det AB = \det A \det B$ $(A, B\ n\times n)$

Pf 1) B not invertible

$\Rightarrow AB$ is not invertible. $0=0$, true

↳ if $C=AB$ is invertible then $C^{-1}AB = I$ B is left inv. $\Rightarrow B$ is inv b/c B is $n\times n$

2) B is invertible

$\Rightarrow B = E_1E_2\cdots E_N$ so $\det AB = \det AE_1E_2\cdots E_N = (\det AE_1\cdots E_{N-1})\det E_N$ etc...

$\det A \cdot \underbrace{\det E_1 \cdots \det E_N}_{\det B}$ so $\det A \det B$

Cor of L1

$\det(AE_1\cdots E_N) = \det A \det E_1 \cdots \det E_N$

Use this corollary to get

$\det AB = \det A \cdot \det E_1 \cdot \ldots \cdot \det E_N$

and $\det B = \det E_1 \cdot \ldots \cdot \det E_N$

$(A = I)$

Thm $\det A = \det A^T$

A is invertible

$A = E_1E_2\cdots E_N$ $\det A = \det E_1 \cdots E_N$

$A^T = E_1^TE_2^T\cdots E_N^T$ $\det A^T = \det E_1^T \cdots E_N^T$

Cofactor expression (row or column expression)

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$$

A_{jk} - A w/o row j and col k

$$\det A = \sum_{k=1}^{n} (-1)^{j+k} a_{jk} \cdot \det A_{jk}$$

$$= \sum_{j=1}^{n} (-1)^{j+k} a_{jk} \cdot \det A_{jk}$$

Ex

$$\begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 3 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} - (0)\begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

or

$$-2\begin{vmatrix} 0 & 2 \\ 2 & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}$$