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#5 on homework

"wrong way"

2×2

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$\vec{v}_1 \quad \vec{v}_2 \quad \vec{v}_3 \quad \vec{v}_4$

$T(A) = A^T \qquad T(\vec{v}_1) = \vec{v}_1 \quad T(\vec{v}_2) = \vec{v}_3 \qquad T(\vec{v}_3) = \vec{v}_2 \quad T(\vec{v}_4) = \vec{v}_4$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

"right way"

$T(A) = \lambda A \qquad A^T = \lambda A$

$$\underset{\substack{\parallel \\ A}}{A^{TT}} = (\lambda A)^T = \lambda A^T = \lambda^2 A$$

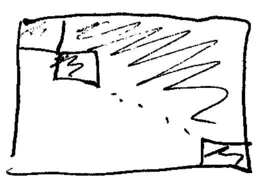
$A = \lambda^2 A$

$I = \lambda^2$

$\lambda = 1, \quad \lambda = -1$

$\lambda = 1 \qquad A^T = A$ symmetric matrix

$\dim = \dfrac{(n+1)(n)}{2}$



$= 1 + 2 + 3 + \dots + n$

$\lambda = -1 \qquad A^T = -A$ anti-symmetric

$a_{jj} = 0$

$\dim = 1 + 2 + \dots (n-1) = \dfrac{(n-1)n}{2}$

$\dim M_{n\times n} = n^2$

$$\frac{(n+1)n}{2} + \frac{(n-1)n}{2} = n^2 \quad \text{matrices are diagonalizable}$$

Necessary and Sufficient condition for diagonalization

Basis of subspaces (AKA direct sums of subspaces)

<u>Def</u>: Let $\vec{V}_1, \vec{V}_2 \dots \vec{V}_r \subset V$

$V_1, V_2 \dots V_r$ is a basis in V iff any $\vec{v} \in V$ admits unique representation $\vec{v} = \vec{v}_1 + \vec{v}_2 + \dots + \vec{v}_r$ where $\vec{v}_k \in V_k$

<u>Ex.</u>

(1) V $\quad \vec{\omega}_1, \vec{\omega}_2 \dots \vec{\omega}_n \to$ basis

Let $V_k = \text{span}(\vec{\omega}_k)$ (One-dimension subspace)

If $\vec{v}_k \in V_k$ then $\exists\, \alpha_k$ s.t. $\vec{v}_k = \alpha_k \vec{\omega}_k$

$$\vec{v} = \sum_{k=1}^{n} \vec{v}_k = \sum_{k=1}^{n} \alpha_k \vec{\omega}_k$$

(2) Let $\vec{b}_1, \vec{b}_2 \dots \vec{b}_n \to$ basis in V

Split $\vec{b}_1, \vec{b}_2 \dots \vec{b}_n$ into r groups

V_k = span of group #k

↳ Essentially the only example of bases of subspaces

<u>Theorem</u>: Let $V_1\ V_2 \dots V_r \subset V$ basis of subspaces

Let B_k be a basis in V_k

Then $\bigcup_{k=1}^{r} B_k$ is a basis in V

↳ take all bases in V_k, lump bases together to get a basis in V

Proof:

Write vectors from B_1 first

Then vectors from B_2 . . .

$\{1, 2, \dots n\} = \sigma_1 \cup \sigma_2 \cup \dots \cup \sigma_r$

$B_k = \{ \vec{b}_j : j \in \sigma_k \}$

Example:

$\underbrace{\bar{b}_1 \ \bar{b}_2 \ \bar{b}_3}_{1} \ \underbrace{\bar{b}_4}_{2} \ \underbrace{\bar{b}_5 \ \bar{b}_6}_{3}$

$\sigma_1 = 1, 2, 3$

$\sigma_2 = 4$

$\sigma_3 = 5, 6$

Take $\vec{v} \in V$

$\vec{v} = \sum_{k=1}^{r} \vec{v}_k$ — unique $\vec{v}_k \in V_k$

$\vec{v}_k = \sum_{j \in \sigma_k} \alpha_j \vec{b}_j$ —unique

$\vec{v} = \sum_k \sum_{j \in \sigma_k} \alpha_j \vec{b}_j = \sum_{j=1}^{n} \alpha_j \vec{b}_j$ — unique

Theorem: A is diagonalizable if $\forall$ e.value λ

alg. mult. = geo. mult

$n = \dim V$ eigen values counting multiplicities

Proof: alg. mult. = geo. mult. for diagonal matrices

Similarity preserves ① characteristic polynomial

① alg. mult.

② dim ker

↳ geom mult.

$V_k = \ker(A - \lambda_k I)$

$V_1 \ V_2 \dots V_r$ — LI → $\sum \vec{v}_k = 0 \quad \vec{v}_k \in V_k \Rightarrow \vec{v}_k = \vec{0} \quad \forall k$

↓ ↓ ↓

$B_1 \cup B_2 \dots \cup B_r$ — LI system of vectors, because # of vectors adds up to n we have a basis