

HW 2/4

Answer key

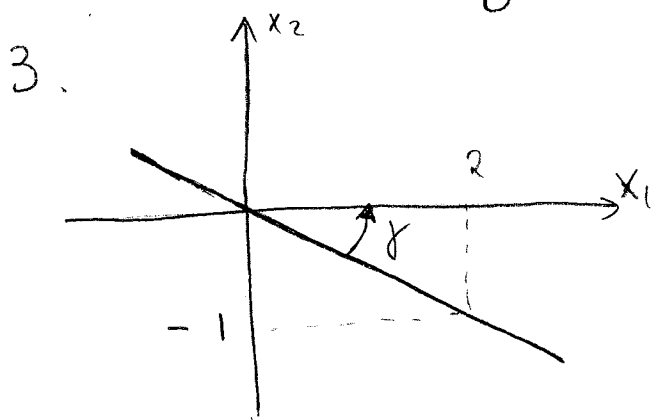
1a  $AB$   $2 \times 3$ ;  $ABD$   $2 \times 1$ ;  $BC^T$   $2 \times 2$   
 $B^TC$   $3 \times 3$ ;  $D^TC^T$   $1 \times 2$

2. 
$$\begin{pmatrix} \cos \gamma & -\sin \gamma \\ \sin \gamma & \cos \gamma \end{pmatrix} \begin{pmatrix} \cos \gamma & \sin \gamma \\ -\sin \gamma & \cos \gamma \end{pmatrix} =$$

$T_\gamma \quad R_\gamma$

$$= \begin{pmatrix} \cos^2 \gamma + \sin^2 \gamma & \cos \gamma \sin \gamma - \sin \gamma \cos \gamma \\ \sin \gamma \cos \gamma - \cos \gamma \sin \gamma & \sin^2 \gamma + \cos^2 \gamma \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and similarly for  $T_\gamma T_\gamma$



$$\cos \gamma = \frac{2}{\sqrt{2^2 + 1^2}} = \frac{2}{\sqrt{5}}$$

$$\sin \gamma = \frac{1}{\sqrt{5}}$$

$$T = R_{-\gamma} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} R_\gamma = \begin{pmatrix} \cos \gamma & \sin \gamma \\ -\sin \gamma & \cos \gamma \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos \gamma & -\sin \gamma \\ \sin \gamma & \cos \gamma \end{pmatrix}$$

$$= \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

$$= \frac{1}{5} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

4. Many possibilities, for example

$$AB = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

$$\text{Then } AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad BA = \begin{pmatrix} 2 & 2 \\ -2 & -2 \end{pmatrix}$$

$$5. \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix}$$

$$= \begin{pmatrix} \cos \alpha \cos \beta - \sin \alpha \sin \beta & -\cos \alpha \sin \beta - \sin \alpha \cos \beta \\ \sin \alpha \cos \beta + \cos \alpha \sin \beta & -\sin \alpha \sin \beta + \cos \alpha \cos \beta \end{pmatrix}$$

Since the result is rotation by  $\alpha + \beta$

$$\begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{pmatrix}$$

we get

$$\begin{aligned} \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \end{aligned}$$