Homework assignment, Jan. 30, 2004.

1. Find a basis in the space of 3×2 matrices $M_{3\times 2}$.

2. True or false:

   (a) Any set containing a zero vector is linearly dependent

   (b) A basis must contain $\mathbf{0}$;

   (c) subsets of linearly dependent sets are linearly dependent;

   (d) subsets of linearly independent sets are linearly independent;

   (e) If $\alpha_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \ldots + \alpha_n\mathbf{v}_n = \mathbf{0}$ then all scalars α_k are zero;

3. Recall, that a matrix is called *symmetric* if $A^T = A$. Write down a basis in the space of *symmetric* 2×2 matrices (there are many possible answers). How many elements are in the basis?

4. Write down a basis for the space of 3×3 *antisymmetric* ($A^T = -A$) matrices. How many elements are in this basis?

5. **(Extra credt)** Is it possible that vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly dependent, but the vectors $\mathbf{w}_1 = \mathbf{v}_1 + \mathbf{v}_2$, $\mathbf{w}_2 = \mathbf{v}_2 + \mathbf{v}_3$ and $\mathbf{w}_3 = \mathbf{v}_3 + \mathbf{v}_1$ are linearly *independent*?