Homework assignment, Feb. 4, 2004.

1. Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & -2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 1 & -1 \end{pmatrix}, \quad D = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$$

a) Mark all the products that are defined, and give the dimensions of the result: AB, BA, ABC, ABD, BC, BC^T, B^TC, DC, D^TC^T.

b) Compute AB, $A(3B+C)$, B^TA, $A(BD)$, $(AB)D$.

2. Let T_γ be the matrix of rotation by γ in $\mathbb{R}^2$. Check by matrix multiplication that $T_\gamma T_{-\gamma} = T_{-\gamma}T_\gamma = I$

3. Find the matrix of the orthogonal projection in $\mathbb{R}^2$ onto the line $x_1 = -2x_2$. **Hint:** What is the matrix of the projection onto the coordinate axis x_1?

You can leave the answer in the form of the matrix product, you do not need to perform the multiplication.

4. Find linear transformations $A, B : \mathbb{R}^2 \to \mathbb{R}^2$ such that $AB = \mathbf{0}$ but $BA \neq \mathbf{0}$.

5. Multiply two rotation matrices T_α and T_β (it is a rare case when the multiplication is commutative, i.e. $T_\alpha T_\beta = T_\beta T_\alpha$, so the order is not essential). Deduce formulas for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ from here.