Homework assignment, Feb. 6, 2004.

Problems 1–7 are easy if you understand what to do (of course, that is the main difficulty!) After you figure out what to do, it takes just a couple of lines to write the solution.

The last problem (# 8) is more complicated. We did similar type of problems for $\mathbb{R}^2$, but it requres some computations and more writing.

1. Find all right inverses to the 1×2 matrix (row) $A = (1, 1)$. Conclude from here that the row A is not left invertible.

2. Find all left inverses of the column $(1, 2, 3)^T$

3. Find two matrices A and B that AB is invertible, but A and B are not. **Hint:** square matrices A and B would not work. **Remark:** It is easy to construct such A and B in the case when the product AB is a 1×1 matrix (a scalar). But can you get 2×2 matrix AB? 3×3? $n \times n$?

4. Suppose the product AB is invertible. Show that A is right invertible and B is left invertible. **Hint:** you can just write formulas for right and left inverses.

5. Let A be $n \times n$ matrix. Prove that if $A^2 = \mathbf{0}$ then A is not invertible

6. Find a non-zero square matrix A such that $A^2 = \mathbf{0}$.

7. Prove, that if $A : V \rightarrow W$ is an isomorphism (i.e. an invertible linear transformation) and $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ is a basis in V, then $A\mathbf{v}_1, A\mathbf{v}_2, \ldots, A\mathbf{v}_n$ is a basis in W.

8. **(Extra Credit)** Find the matrix of the rotation in $\mathbb{R}^3$ by the angle α around the vector $(1, 2, 3)^T$. We assume that rotation is counterclockwise if we sit at the tip of the vector and looking at the origin.

You can present the answer as a product of several matrices: you don't have to perform the multiplications.