Homework assignment, Feb. 9, 2004.

To be collected Wednesday, Feb. 11

1. Suppose $AB = \mathbf{0}$ for some non-zero matrix B. Can A be invertible? Justify.

2. Write matrices of the linear transformations T_1 and T_2 in $\mathbb{R}^5$, defined as follows: T_1 interchanges the coordinates x_2 and x_4 of the vector $\mathbf{x}$, and T_2 just adds to the coordinate x_2 a times the coordinate x_4, and does not change other coordinates, i.e.

$$T_1 \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_4 \\ x_3 \\ x_2 \\ x_5 \end{pmatrix}, \qquad T_2 \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 + ax_4 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix};$$

here a is some fixed number.

Show that T_1 and T_2 are invertible transformations, and write the matrices of the inverses. **Hint:** it may be simpler, if you first describe the inverse transformation, and then find its matrix, rather than trying to guess (or compute) the inverses of the matrices T_1, T_2.

3. Multiplication of a 2-vector by an arbitrary 2×2 matrix usually requires 4 multiplications.

Suppose a 2×1000 matrix D contains coordinates of 1000 points in $\mathbb{R}^2$. How many multiplications is required to transform these points using 2 arbitrary 2×2 matrices A and B. Compare 2 possibilities, $A(BD)$ and $(AB)D$.

4. Write 4×4 matrix performing perspective projection to x-y plane with center $(d_1, d_2, d_3)^T$.

5. Give examples of matrices (say 2×2) such that:

   a) $A + B$ is not invertible although both A and B are invertible;

   b) $A + B$ is invertible although both A and B are not invertible;

   c) All of A, B and $A + B$ are invertible

6. Let A be an invertible simmetric ($A^T = A$) matrix. Is the inverse of A symmetric? Justify.

7. **(The hard one)** A transformation T in $\mathbb{R}^3$ is a rotation about the line $y = x + 3$ in x-y plane through an angle γ. Write 4×4 matrix corresponding to this transformation. You can leave the result as a product of matrices.