Homework assignment, Feb. 11, 2004.

1. Write the systems of equations below in matrix form and as vector equations:

a) $$\begin{cases} x_1 + 2x_2 - x_3 = -1 \\ 2x_1 + 2x_2 + x_3 = 1 \\ 3x_1 + 5x_2 - 2x_3 = -1 \end{cases}$$

b) $$\begin{cases} x_1 - 2x_2 - x_3 = 1 \\ 2x_1 - 3x_2 + x_3 = 6 \\ 3x_1 - 5x_2 = 7 \\ x_1 + 5x_3 = 9 \end{cases}$$

c) $$\begin{cases} x_1 - 4x_2 - x_3 + x_4 = 3 \\ 2x_1 - 8x_2 + x_3 - 4x_4 = 9 \\ -x_1 + 4x_2 - 2x_3 + 5x_4 = -6 \end{cases}$$

2. Solve the systems from the previous problem. Write the answers in the vector form.

3. Find *all* solutions of the vector equation

$$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{0},$$

where $\mathbf{v}_1 = (1,1,0)^T$, $\mathbf{v}_2 = (0,1,1)^T$ and $\mathbf{v}_3 = (1,0,1)^T$. What conclusion can you make about linear independence (dependence) of the system of vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$?

4. Prove Theorem 4.1, i.e. prove that $\operatorname{trace}(AB) = \operatorname{trace}(BA)$ whenever both products are defined.