Homework assignment, Feb. 13, 2004.

To be collected Mon, Feb. 16

1. For what value of b the system

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ 4 \\ b \end{pmatrix}$$

has a solution. Find the general solution of the system for this value of b.

2. Solve linear system

$$\begin{cases} x_1 + 2x_2 - x_3 + 3x_4 = 2 \\ 2x_1 + 4x_2 - x_3 + 6x_4 = 5 \\ x_2 + 2x_4 = 3 \end{cases}$$

3. Determine, which of the following systems of vectors are bases in $\mathbb{R}^3$:

   a) $(1, 2, -1)^T$, $(1, 0, 2)^T$, $(2, 1, 1)^T$;

   b) $(-1, 3, 2)^T$, $(-3, 1, 3)^T$, $(2, 10, 2)^T$;

   c) $(67, 13, -47)^T$, $(\pi, -7.84, 0)^T$, $(3, 0, 0)^T$.

4. Do the polinomials $x^3 + 2x_9$, $x^2 + x + 1$, $x^3 + 5$ generate (span) $\mathbb{P}_3$? Justify your answer

5. Can 5 vectors in $\mathbb{R}^4$ be linearly independent? Justify your answer.

6. Find the inverse of the matrix.

$$\begin{pmatrix} 1 & 2 & 1 \\ 3 & 7 & 3 \\ 2 & 3 & 4 \end{pmatrix}$$

Show all steps

7. Show, that if the equation $A\mathbf{x} = \mathbf{0}$ has unique solution (i.e. if echelon form of A has pivot in every column) then A is left invertible. **Hint:** elementary matrices may help.

Note: It was shown in the text, that if A is left invertible, then the equation $A\mathbf{x} = \mathbf{0}$ has unique solution. But here you are asked to prove the converse of this statement, which was not proved.