Homework assignment, Feb. 25, 2004.

Due Friday, 2/27 (collected)

All problems except #4 and #5 require only minimal amount of computations.

1. True or false

a) Every change of coordinate matrix is square.

b) Every change of coordinate matrix is invertible.

c) The matrices A and B are called similar if $B = Q^T AQ$ for some matrix Q.

d) The matrices A and B are called similar if $B = Q^{-1}AQ$ for some matrix Q.

e) Similar matrices do not need to be square.

2. Check that the system of vectors

$$(1, 2, 1, 1)^T, \quad (0, 1, 3, 1)^T, \quad (0, 3, 2, 0)^T, \quad (0, 1, 0, 0)^T.$$

is a basis in $\mathbb{R}^4$. Try to do minimal amount of computations.

3. Find the change of coordinate matrix that changes the coordinates in the basis

$$(1, 2, 1, 1)^T, \quad (0, 1, 3, 1)^T, \quad (0, 3, 2, 0)^T, \quad (0, 1, 0, 0)^T.$$

to the standard coordinates in $\mathbb{R}^4$ (i.e. to the coordinates in the standard basis $\mathbf{e}_1, \ldots, \mathbf{e}_4$).

4. Find the change of coordinates matrix that changes the coordinates in the basis $1, 1 + t$ in $\mathbb{P}_1$ to the coordinates in the basis $1 - t, 2t$.

5. Let T be the linear operator in $\mathbb{R}^2$ defined (in the standard coordinates) by

$$T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x + y \\ x - 2y \end{pmatrix}$$

Find the matrix of T in the standard basis and in the basis

$$(1, 1)^T, \qquad (1, 2)^T.$$

6. Prove, that if A and B are similar matrices then $\operatorname{trace} A = \operatorname{trace} B$. **Hint:** recall how $\operatorname{trace}(XY)$ and $\operatorname{trace}(YX)$ are related.

7. Are the matrices

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \qquad \text{and} \qquad \begin{pmatrix} 0 & 2 \\ 4 & 2 \end{pmatrix}$$

similar? Justify.

8. **A modified problem from the previous assignment:** Prove or disprove: If the columns of a square ($n \times n$) matrix A are linearly independent, so are the rows of $A^3 = AAA$.