Homework assignment, Feb. 27, 2004.

Read Sections 1–3 of Chapter 3. Skip proofs of Theorems 3.4, 3.5 (we will go over them next class).

1. If A is an $n \times n$ matrix, how the determinants $\det A$ and $\det(5A)$ are related? **Remark:** $\det(5A) = 5 \det A$ only in the trivial case of 1×1 matrices

2. How the determinants $\det A$ and $\det B$ are related if

a)

$$A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}, \qquad B = \begin{pmatrix} 2a_1 & 3a_2 & 5a_3 \\ 2b_1 & 3b_2 & 5b_3 \\ 2c_1 & 3c_2 & 5c_3 \end{pmatrix}.$$

b)

$$A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}, \qquad B = \begin{pmatrix} 3a_1 & 4a_2 + 5a_1 & 5a_3 \\ 3b_1 & 4b_2 + 5b_1 & 5b_3 \\ 3c_1 & 4c_2 + 5c_1 & 5c_3 \end{pmatrix}.$$

In the following two problems you can use the fact that $\det A = \det A^T$, so you can use either column or row operations to compute determinants.

3. Using column or row operations compute the determinants

$$\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{vmatrix}, \qquad \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}, \qquad \begin{vmatrix} 1 & 0 & -2 & 3 \\ -3 & 1 & 1 & 2 \\ 0 & 4 & -1 & 1 \\ 2 & 3 & 0 & 1 \end{vmatrix}, \qquad \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix}$$

4. A square ($n \times n$) matrix is called skew-symmetric (or antisymmetric) if $A^T = -A$. Prove that if A is skew-symmetric and n is odd, then $\det A = 0$. Is this true for even n?

In the following problem you can use the fact that $\det(AB) = \det A \det B$.

5. A square matrix is called *nilpotent* if $A^k = \mathbf{0}$ for some positive integer k. Show that for a nilpotent matrix A $\det A = 0$.

6. Suppose a matrix M can be written in a block triangular form

$$\begin{pmatrix} A & B \\ \mathbf{0} & C \end{pmatrix}$$

where A and B are square matrices. Show that $\det M = \det A \det C$.