Homework assignment, March 1, 2004.

Due Wednesday, 3/3 (collected)

1. Prove that if the matrices A and B are similar, than $\det A = \det B$.

2. A square matrix Q is called orthogonal if $Q^T Q = I$. Prove that if Q is an orthogonal matrix then $\det Q = \pm 1$.

3. Evaluate the determinants using any method

$$\begin{vmatrix} 0 & 1 & 1 \\ 1 & 2 & -5 \\ 6 & -4 & 3 \end{vmatrix}, \qquad \begin{vmatrix} 1 & -2 & 3 & -12 \\ -5 & 12 & -14 & 19 \\ -9 & 22 & -20 & 31 \\ -4 & 9 & -14 & 15 \end{vmatrix}$$

4. Use row (column) expansion to evaluate the determinants). Note, that you don't need to use the first row (column): picking row (column) with many zeroes will simplify your calculations.

$$\begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 5 \\ 1 & -3 & 0 \end{vmatrix}, \qquad \begin{vmatrix} 4 & -6 & -4 & 4 \\ 2 & 1 & 0 & 0 \\ 0 & -3 & 1 & 3 \\ -2 & 2 & -3 & -5 \end{vmatrix}$$

5. Show that

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (z-x)(z-y)(y-x).$$

This is a particular case of the so-called Vandermonde determinant.

6. For the $n \times n$ matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & a_0 \\ -1 & 0 & 0 & \dots & 0 & a_1 \\ 0 & -1 & 0 & \dots & 0 & a_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & a_{n-2} \\ 0 & 0 & 0 & \dots & -1 & a_{n-1} \end{pmatrix}$$

compute $\det(A + tI)$, where I is $n \times n$ identity matrix. You should get a nice expression involving $a_0, a_1, \dots, a_{n-1}$ and t. Row expansion and induction is probably the best way to go.

7. (A problem from the previous assignment) How the determinants $\det A$ and $\det B$ are related if

a)

$$A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}, \qquad B = \begin{pmatrix} 2a_1 & 3a_2 & 5a_3 \\ 2b_1 & 3b_2 & 5b_3 \\ 2c_1 & 3c_2 & 5c_3 \end{pmatrix}.$$

b)

$$A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}, \qquad B = \begin{pmatrix} 3a_1 & 4a_2 + 5a_1 & 5a_3 \\ 3b_1 & 4b_2 + 5b_1 & 5b_3 \\ 3c_1 & 4c_2 + 5c_1 & 5c_3 \end{pmatrix}.$$