Homework assignment, March 12, 2004.

Due Monday, 3/15 (collected)

1. Find characteristic polynomials, eigenvalues and eigenvectors of the following matrices:

$$\begin{pmatrix} 4 & -5 \\ 2 & -3 \end{pmatrix}, \qquad \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{pmatrix}.$$

When it is possible, diagonalize the matrices, i.e. write them in a form $A = SDS^{-1}$, where D is a diagonal matrix. Do not compute S^{-1}.

2. Compute eigenvalues and eigenvectors of the rotation matrix

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}.$$

Note, that the eigenvalues (and eigenvectors) do not need to be real.

3. Let

$$A = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}.$$

Find A^{2004} by diagonalizing A.

4. True or false:

a) Every linear operator in an n-dimensional vector space has n distinct eigenvalues.

b) If a matrix has one eigenvector, it has infinitely many eigenvectors.

c) There exists a square real matrix with no real eigenvalues.

d) There exists a square matrix with no (complex) eigenvectors.

e) Similar matrices always have the same eigenvalues.

f) Similar matrices always have the same eigenvectors.

g) The sum of two eigenvectors of a matrix A is always an eigenvector

h) The sum of two eigenvectors of a matrix A corresponding to the same eigenvalue is always an eigenvector

5. Construct a matrix A with eigenvalues 1 and 3 and corresponding eigenvectors $(1, 2)^T$ and $(1, 1)^T$. Is such matrix unique?