Homework assignment, March 15, 2004.

See if you can do Exercise 1.4 in Ch. 4.

1. Diagonalize the following matrices, if possible:

a) $\begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$.

b) $\begin{pmatrix} -1 & -1 \\ 6 & 4 \end{pmatrix}$.

c) $\begin{pmatrix} -2 & 2 & 6 \\ 5 & 1 & -6 \\ -5 & 2 & 9 \end{pmatrix}$ ($\lambda = 2$ is one of the eigenvalues)

2. Find all square roots of the matrix

$$A = \begin{pmatrix} 5 & 2 \\ -3 & 0 \end{pmatrix}$$

i.e. find all matrices B such that $B^2 = A$. **Hint:** Finding a square root of a diagonal matrix is easy. You can leave your answer as a product.

3. Let A be $n \times n$ matrix. True or false:

a) A^T has the same eigenvalues as A.

b) A^T has the same eigenvectors as A.

c) If A is is diagonalizable, then so is A^T.

Justify your conclusions.

4. Let A be a 3×3 matrix with eigenvalues $1, 2, 5$. What is $\operatorname{trace}(A^3)$, $\det(A^2 + 2A - 2I)$?

5. a) Consider the transformation T in the space $M_{2\times 2}$ of 2×2 matrices, $T(A) = A^T$. Find all its eigenvalues and eigenvectors. Is it possible to diagonalize this transformation? **Hint:** While it is possible to write a matrix of this linear transformation in some basis, compute characteristic polynomial, and so on..., it is easier to find eigenvalues and eigenvectors directly from the definition.

b) Can you do the same problem but in the space of $n \times n$ matrices?