Homework assignment, March 19, 2004.

Due Monday, 3/22 (collected)

1. Compute

$$(3+2i)(5-3i), \qquad \frac{2-3i}{1-2i}, \qquad \operatorname{Re}\left(\frac{2-3i}{1-2i}\right), \qquad (1+2i)^3, \qquad \operatorname{Im}((1+2i)^3)$$

2. For vectors $\mathbf{x} = (1, 2i, 1+i)^T$ and $\mathbf{y} = (i, 2-i, 3)^T$ compute

   a) $(\mathbf{x}, \mathbf{y})$, $\|\mathbf{x}\|^2$, $\|\mathbf{y}\|^2$, $\|\mathbf{y}\|$.

   b) $(3\mathbf{x}, 2i\mathbf{y})$, $(2\mathbf{x}, i\mathbf{x} + 2\mathbf{y})$.

   c) $\|\mathbf{x} + 2\mathbf{y}\|$

Remark: After you have done part a), you can do parts b) and c) without actually computing all vectors involved, just by using the properties of inner product.

3. Prove that for vectors in a inner product space

$$\|\mathbf{x} \pm \mathbf{y}\|^2 = \|\mathbf{x}\|^2 + \|\mathbf{y}\|^2 \pm 2\operatorname{Re}(\mathbf{x}, \mathbf{y})$$

Recall that $\operatorname{Re} z = \frac{1}{2}(z + \overline{z})$

4. Explain why each of the following is not an inner product on a given vector space;

   a) $(\mathbf{x}, \mathbf{y}) = x_1 y_1 - x_2 y_2$ on $\mathbb{R}^2$.

   b) $(A, B) = \operatorname{trace}(A + B)$ on the space of real 2×2 matrices.

   c) $(f, g) = \int_0^1 f'(t)\overline{g(t)}dt$ on the space of polynomials; $f'(t)$ denotes derivative.

5. Prove the parallelogram identity for an inner product space V,

$$\|\mathbf{x} + \mathbf{y}\|^2 + \|\mathbf{x} - \mathbf{y}\|^2 = 2(\|\mathbf{x}\|^2 + \|\mathbf{y}\|^2).$$

6. Let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ be a spanning set (in particular a basis) in an inner product space V. Prove that

   a) If $(\mathbf{x}, \mathbf{v}) = 0$ for all $\mathbf{v} \in V$, then $\mathbf{x} = \mathbf{0}$.

   b) If $(\mathbf{x}, \mathbf{v}_k) = 0$ $\forall k$, then $\mathbf{x} = \mathbf{0}$.

   c) If $(\mathbf{x} - \mathbf{y}, \mathbf{v}_k) = 0$ $\forall k$, then $\mathbf{x} = \mathbf{y}$