Homework assignment, March 24, 2004.

1. Find the orthogonal projection of a vector $(1,1,1,1)^T$ onto the subspace spanned by the vectors $\mathbf{v}_1 = (1,3,1,1)^T$ and $\mathbf{v}_2 = (2,-1,1,0)^T$ (note that $\mathbf{v}_1 \perp \mathbf{v}_2$).

2. Find the distance from a vector $(1,2,3,4)$ to the subspace spanned by the vectors $\mathbf{v}_1 = (1,-1,1,0)^T$ and $\mathbf{v}_2 = (1,2,1,1)^T$ (note that $\mathbf{v}_1 \perp \mathbf{v}_2$). Can you find the distance without actually computing the projection? That would simplify the calculations.

3. True or false: if E is a subspace of V, then $\dim E + \dim(E^\perp) = \dim V$? Justify.

4. Let P be the orthogonal projection onto subspace E of an inner product space V, $\dim V = n$, $\dim E = r$. Find eigenvalues, eigenvectors (eigenspaces). Find algebraic and geometric multiplicities of each eigenvalue.

5.
   a) Find the matrix of the orthogonal projection onto one-dimensional subspace in $\mathbb{R}^n$ spanned by the vector $(1,1,\dots,1)^T$.
   b) Let A be the $n \times n$ matrix with all entries equal 1. Compute its eigenvalues and their multiplicities (use previous problem).
   c) Compute eigenvalues (and multiplicities) of the matrix $A - I$, i.e. of the matrix with zeroes on the main diagonal and ones everywhere else.
   d) Compute $\det(A - I)$.

6. Read s. 1.5. Consider space $\mathbb{R}^2$ with the norm $\|\cdot\|_p$, introduced in this section. For $p = 1, 2, \infty$ draw the "unit ball" B_p in the norm $\|\cdot\|_p$

$$B_p := \{\mathbf{x} \in \mathbb{R}^2 : \|\mathbf{x}\|_p \le 1\}.$$

Can you guess how the balls B_p for other p look like?