Homework assignment, March 26, 2004.

Read s. 3.1 (Gram Schmidt orthogonalization), 3.2, 3.3

1. Apply Gram–Schmidt orthogonalization to the system of vectors $(1, 2, -2)^T$, $(1, -1, 4)^T$, $(2, 1, 1)^T$

2. Apply Gram–Schmidt orthogonalization to the system of vectors $(1, 2, 3)^T$, $(1, 3, 1)^T$. Write the matrix of orthogonal projection onto 2-dimensional subspace spanned by these vectors.

3. Complete an orthogonal system obtained in the previous problem to an orthogonal basis in $\mathbb{R}^3$, i.e. add to the system some vectors (how many?) to get an orthogonal basis.

Can you describe how to complete an orthogonal system to an orthogonal basis in general situation of $\mathbb{R}^n$ or $\mathbb{C}^n$?

4. Find the distance from a vector $(2, 3, 1)^T$ to the subspace spanned by the vectors $(1, 2, 3)^T$, $(1, 3, 1)^T$. Note, that I am only asking to find the distance to the subspace, not the orthogonal projection.

5 *(Legendre's polynomials:)*. Let inner product on the space of polynomials be defined by $(f, g) = \int_{-1}^{1} f(t)\overline{g(t)}dt$. Apply Gram-Schmidt orthogonalization to the system $1, t, t^2, t^3$.

Legendre's polynomials are particular case of the so-called orthogonal polynomials, which play an important role in many branches of mathematics.

6. Let $P = P_E$ be the matrix of an orthogonal projection onto a subspace E. Show that

a) The matrix P is *self-adjoint*, meaning that $P^* = P$.

b) $P^2 = P$.

Remark: above 2 properties completely characterize orthogonal projection, i.e. any matrix P satisfying these properties is the matrix of some orthogonal projection. We will discuss this some time later.

7. For a set $E \subset V$ its orthogonal complement $E^\perp$ consists of all vectors orthogonal to E. Show that if E is a subspace,then $(E^\perp)^\perp = E$.
Hint: Take $\mathbf{x}$ orthogonal to $E^\perp$ and see what is its orthogonal projection onto E