Homework assignment, April 26, 2004.

Due Wednesday, 4/28 (collected)

1. Find singular value decomposition $A = W\Sigma V^*$ where V and W are unitary matrices for the following matrices

a) $A = \begin{pmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{pmatrix}$

b) $A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$

2. Find singular value decomposition of the matrix

$$A = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$$

Find

a) $\max_{\|\mathbf{x}\| \le 1} \|A\mathbf{x}\|$ and the vectors where the maximum is attained.

b) $\min_{\|\mathbf{x}\| \le 1} \|A\mathbf{x}\|$ and the vectors where the minimum is attained.

c) the image $A(B)$ of the closed unit ball in $\mathbb{R}^2$, $B = \{\mathbf{x} \in \mathbb{R}^2 : \|\mathbf{x}\| \le 1\}$. Describe $A(B)$ geometrically.

3. True or false

a) Singular values of a matrix are also eigenvalues of the matrix.

b) Singular values of a matrix A are eigenvalues of A^*A.

c) Is s is a singular value of a matrix A and c is a scalar, then $|c|s$ is a singular value of cA.

d) The singular values of any linear operator are non-negative.

e) Singular values of a self-adjoint matrix coincide with its eigenvalues.

4. Let A be an $m \times n$ matrix. Prove that *non-zero* eigenvalues of the matrices A^*A and AA^* (counting multiplicities) coincide.

Can you say when zero eigenvalue of A^*A and zero eigenvalue of AA^* have the same multiplicity?

5. Let s be the largest singualr value of an operator A, and let λ be the eigenvalue of A with largest absolute value. Show that $|\lambda| \leq s$.