Homework assignment, May 3, 2004.

1. Using Silvester's Criterion of Positivity check if the matrices

$$A = \begin{pmatrix} 4 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 3 & -1 & 2 \\ -1 & 4 & -2 \\ 2 & -2 & 1 \end{pmatrix}$$

is positive definite or not.

Are the matrices $-A$, A^3 and A^{-1}, $A + B^{-1}$, $A + B$, $A - B$ positive definite?

2. True or false:

a) If A is positive definite, then A^5 is positive definite

b) If A is negative definite, then A^8 is negative definite

c) If A is negative definite, then A^{12} is positive definite.

d) If A is positive definite and B is negative semidefinite, then $A - B$ is positive definite

e) If A is indefinite, and B is positive definite, then $A + B$ is indefinite.

3 *(A review problem).* Prove, that if E and F are subspaces of $\mathbb{R}^n$ and $\dim E + \dim F > n$, then $E \cap F \neq \{\mathbf{0}\}$. i.e. that there exists a vector $\mathbf{x} \neq \mathbf{0}$ such that $\mathbf{x} \in E \cap F$