

Demo Help Manual: A Guide to The Demos

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Abstract

Demos are an integral part of the Differential Geometry course, MA 106. An understanding of the demos and how they work is essential to visualize and learn topics in differential geometry. Through the course the demos range from simple to extremely difficult. Knowing how to manipulate the demos through entering new functions to designing your own demo is vital for this course. This guide should help you understand the basic and more complicated aspects of the demos.

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1 Control Window

1.1 Demo drop-down menu

The windows are arranged as a hierarchy. Closing one window will close others based on the location in the hierarchy. For example, closing the Controls window closes the entire demo whereas closing the 2D graph window closes the 2D graph window and any windows opened from it.

Show Applet Tag Opens a HTML fragment which brings up the code for the demo in its current state. The code may be copied and pasted into appropriate sections on the course website for posting homeworks, discussions or exams. *Show Applet Tag* is the way to save your demo. On a UNIX machine, to copy the data, highlight all of the text in the dialogue box and in an open shell click on the middle button on the mouse. This will copy all of the selected text to the new location. Select *Close* at the bottom of the window to close the window.

Import Data ... Opens a text box in which DATA code may be posted. Select the *OK* button at the bottom of the window to compile and display entered DATA code.

New Demo Closes and deletes current demo and opens blank Control Window.

Exit Demo Closes the current demo. Please note: the data in the demo is lost when the window is closed unless the user specifically saves the applet tag.

1.2 Controls drop-down menu

Add New Function Opens an Add Function window. In the text box type the desired function following the noted syntax: form name (variables) = definition. Example: $X(u,v) = u,v, u^2 + v^2$.

Note: form name and variables are case sensitive, X and x are viewed as different Select *OK* or press the return key and the function will be added to the Controls window.

Add New Interval Opens an Add Interval window. In the text box type the desired interval following the noted syntax: form name = start, end, resolution. Example: $a = -1, 1, 10$. Select *OK* or press the return key and the interval will be added to the Controls window. It is

important to add the same number of intervals as there are variables in the function to be graphed.

Add New Variable Opens an Add Variable window. In the text box type the desired variable following the noted syntax: form name = minimum, maximum, steps. Example: $x = 0, 2\pi, 2\pi$. Select *OK* or press the return key and the variable will be added to the Controls window. The variable will be added to the control window with a slider that has both a single and double arrow in each direction. Clicking on the double arrow moves the variable through each step to the max or min depending upon the direction selected.

Add New Constant Opens an Add Constant window. In the text box type the desired constant following the noted syntax: form name = definition. Example: $P = 10, 0, 2\pi$. The new constant does not have to be an integer or point value, the new constant may be defined as an already assigned variable, $P = x$. Select *OK* or press the return key and the variable will be added to the Controls window.

Add New Readout Adds a Readout section to the Controls Window. The Readout section is two text boxes, the first allows data entry and the other gives the readout when return is pressed. Example: If $X(1,2)$ is typed in the left-hand box, the point defined by $X(1,2)$ will appear in the right-hand readout box. If the function $X(u,v)$ is changed the readout will also change accordingly.

New 2D Graph Opens a new window with the x and y-axis. The 2D Graph has four drop-down menus *Tools, Plot, View and Window*. See note regarding window heirarchy.

New 3D Graph Opens a new window with the x, y and z-axis. The 3D Graph has four drop-down menus *Tools, Plot, View and Window*.

Show/Hide Items Opens a text box that lists all of the functions, variables, constants, etc that have been created. Each item may be selected to show or hide in the Control window. To simplify viewing and reduce options for users hide all non-essential items. To show items, select *Item is Visible*.

2 2D Graph Window

2.1 Tools drop-down menu

Translate Translates the axis within the 2D graph window. Holding the Alt (or Option) key while using the mouse also functions to translate the axis.

Zoom Zooms in and out. Movement of the mouse vertically toward the top of the screen zooms in while vertical movement toward the bottom of the screen zooms out. Holding the shift key while moving the mouse also serves to zoom in and out.

Point Cursor is able to click on *Hotspot* (see below) and move the point with the movement of the cursor. The point is dropped when left mouse button is released.

2.2 Plot drop-down menu

Add Plot Lists seven options for possible types of Plots. The options are:

1. Point
2. Vector
3. Curve
4. Wireframe
5. Polygon
6. Polyhedron
7. Field

Prioritize Plots May be selected on or off. Prioritize plots has no effect unless using the *Out of screen adjustment* under *More Options ...* in the plot dialog. When selected, prioritize plots draws the plots consecutively and allows for the drawing to prevent conflicting and overlapping figures. It is similar to choosing between *Sort* and *No Sorting or Buffering* styles from the styles menu of a 3D graph window.

Show/Hide in Menu ... Similar to *Show/Hide Items* in Control Window Controls drop-down menu. Allows demo designer to make each item visible or invisible to the user.

2.3 View drop-down menu

Zoom In Zooms in on center of 2D graph window.

Zoom Out Zooms out from center of window.

2.4 Window drop-down menu

Close Window Closes 2D graph window and all windows opened from the 2D graph drop-down menus.

Window Title ... Enter new window title in text box. Pressing return key or *Ok* in window closes the window and new title is displayed at the top of the 2D graph window.

Set Point Variables ... One of several ways to set mouse interactions into a graph. Enter an already defined variable into the desired fields. Movement of the mouse, either clicking or drgagging, sets the variable to the mouse location. This method is outdated and is not recommended. It is primarily used to convert previous demos to the current software.

Edit Hotspots A hotspot is an invisible point that can be selected and permanently moved by clicking and dragging with the mouse. Hotspots are only available for 2D Graphs through the 2D Graph window.

To add a hotspot, select *Add Hotspot*. Enter a name for the hotspot (ex. A), the origin for the hotspot and current point, which is the current location of the hotspot. The origin (0,0) is often beneficial to set at the point origin and current point because the point is invisible.

Set Point to may be used to constrain the hotspot. The variable *Point* refers to the location of the user's mouse. For example, to constrain the hotspot to the unit circle enter `unit(Point)` and the location of the hotspot will be set to the point at the unit vector of the user's mouse location. Note: anything in *Set Point to* is centered at the origin specified for the hotspot.

To view the Hotspot, it is possible to define a point that moves with the Hotspot and allows for easy detection and movement. In the Plot drop-down menu select *Add Plot* and from the side menu select *Add Point...* In the text box to enter a point insert the name of the Hotspot (ex. A) and press the enter key. A dot will appear at the location of the

point. To move the point, select *Point* in the Tools drop-down menu in the 2D Graph window. Click and drag the point to any location.

To edit a hotspot, highlight the desired hotspot from the list of hotspots. Select *Edit ...* . An editing window opens and all aspects of the hotspot may be edited.

3 3D Graph Window

3.1 Tools drop-down menu

Rotate Rotates the axis with the origin fixed. Holding the control key while moving the mouse rotates the axis.

Translate Translates the axis within the 3D graph window. Holding the Alt (or Option) key while using the mouse also functions to translate the axis.

Zoom Zooms in and out. Movement of the mouse vertically toward the top of the screen zooms in while vertical movement toward the bottom of the screen zooms out. Holding the shift key while moving the mouse also serves to zoom in and out.

Point Cursor is able to click on *Hotspot* (see below) and move the point with the movement of the cursor. The point is dropped when left mouse button is released.

3.2 Plot drop-down menu

Add Plot Lists eight options for possible types of Plots. The options are:

1. Point
2. Vector
3. Curve
4. Surface
5. Wireframe
6. Polygon
7. Polyhedron
8. Field

Show/Hide in Menu . . . Similar to *Show/Hide Items* in Control Window Controls drop-down menu. Allows demo designer to make each item visible to user.

3.3 Styles drop-down menu

Open Surfaces Creates a wireframe of the surface.

Filled Surfaces Fills in the wireframe of the surface with color. The wireframe lines are still drawn.

Filled Surfaces, No Lines Fills in the wireframe of the surface with color and removes the wireframe lines.

No Sorting or Buffering Sorting and Buffering are two different drawing algorithms. By selecting *No Sorting or Buffering* no drawing algorithm is selected. The drawing and movement is much faster however the drawing quality is lacking.

Sort The sorting algorithm sorts all the polygons in the graph according the minimum out-of-screen coordinate. The polygons are subsequently drawn from back to front. Due to the lack of per-pixel comparisons, intersections are often poorly drawn in the Sort algorithm.

Z Buffer Z Buffer is the most accurate drawing algorithm. However, it is also the slowest drawing algorithm and machines can be quite slow when Z Buffer is selected. The Z Buffer algorithm compares the out-of-screen, z, coordinate of each pixel and draws the pixel. This algorithm is quite accurate for intersections and overlapping regions of graphs.

Use Lighting Lighting three-dimensional plots allows for the viewer to see shadow and thus differentiate regions of the graph. Without lighting, when the Use Lighting box is unchecked, the figure is a solid color and three-dimensionality is visible only at the edges of the graphed surface.

Suspend While Dragging Suspending while dragging allows for the quick movement of the plots. The figures are converted to open surfaces for the duration of the movement and then redrawn as filled surfaces, with or without lines depending upon the user's selection.

3.4 View drop-down menu

Zoom In Zooms in on center of 3D graph window.

Zoom Out Zooms out from center of window.

Default Default view of the axis. Spacebar also brings the surface back to the default view.

Up X Axis View of the 3D plot up the x-axis, displays the y-z plane in the window. Typing x also views the surface up the x-axis.

Down X Axis View of the 3D plot down the x-axis. Typing shift + x also views the surface down the x-axis.

Up Y Axis View of the 3D plot up the y-axis. Typing y also views the surface up the y-axis.

Down Y Axis View of the 3D plot down the y-axis. Typing shift-y also views the surface down the y-axis.

Up Z Axis View of the 3D plot up the z-axis. Typing z also views the surface up the z-axis.

Down Z Axis View of the 3D plot down the z-axis. Typing shift-z also views the surface down the z-axis.

3.5 Window drop-down menu

Close Window Closes 2D graph window and all windows opened from the 2D graph drop-down menus.

Window Title ... Enter new window title in text box. Pressing return key or *Ok* in window closes the window and new title is displayed at the top of the 2D graph window.

Set Point Variables ... One of several ways to set mouse interactions into a graph. Enter an already defined variable into the desired fields. Movement of the mouse, either clicking or drgagging, sets the variable to the mouse location. This method is outdated and is not recommended. It is primarily used to convert previous demos to the current software.

Link Rotation ... Allows the user to link two or more graphs so that when one is selected and rotated all graphs rotate similarly.

4 Add Plot

4.1 Point

Enter a point Enter the location of the point. The point may be defined by hotspots, variables or equations.

Coloring Coloring works as layers. Each plot may have numerous colors layered over each other. Opacity and blending allow the visibility of multiple color schemes concurrently. See *Coloring* section for more details.

Plot is Visible The point may be hidden by unchecking *Plot is Visible*. This function serves to preserve all of the plot settings but prevents the drawing of the plot.

Style There are several options for the style of the point: Dot, Cross, Sphere, None. In the 2D Graph the Sphere point style does not look ideal.

Size The size of the dot can be varied on a scale of 1 to 25. The default size setting is 10.

Show Label The point may be labeled in the graph. Click *Show Label* and select the text box and enter the desired label. The label appears to the lower-right of the point.

More Options . . . The Out of screen adjustment and Transparency Threshold are used when multiple plots are drawn over each other. These options are to prevent the confusion of intersections and other drawing dilemmas that arise from overlapping plots.

Remove Plot Selecting remove plot deletes the plot from the graph window and closes the plot window.

4.2 Vector

Vector and Base point Enter the end point and base point of the vector, these may be constants, variables or functions or any combination. The program draws a vector connecting the two defined points in the direction of the *Vector*.

Coloring Coloring works as layers. Each plot may have numerous colors layered over each other. Opacity and blending allow the visibility of multiple color schemes concurrently. See *Coloring* section for more details.

Plot is Visible The vector may be hidden by unchecking *Plot is Visible*.

Draw Thick The thickness of the vector may be increased or decreased by checking or unchecking the *Draw Thick* box respectively.

Length The default setting for the relative length of the vector is 1.0. By increasing or decreasing the length, the vector will be scaled by the value in the dialogue box to the right of the sliding bar.

End Point Style and Size There are seven options for the end point style. The size of the end point is variable to change the emphasis on the point.

Base Point Style and Size There are seven options for the base point style. The size of the base point is variable to change the emphasis on the point.

More Options ... The Out of screen adjustment and Transparency Threshold are used when multiple plots are drawn over each other. These options are to prevent the confusion of intersections and other drawing dilemmas that arise from overlapping plots.

Remove Plot Selecting remove plot deletes the plot from the graph window and closes the plot window.

4.3 Curve

Enter parametric curve In the dialogue box, enter the desired curve. Curves must be dependent on exactly one interval.

Coloring Coloring works as layers. Each plot may have numerous colors layered over each other. Opacity and blending allow the visibility of multiple color schemes concurrently. See *Coloring* section for more details.

Plot is Visible The curve may be hidden by unchecking *Plot is Visible*.

Draw Thick The thickness of the curve may be increased or decreased by checking or unchecking the *Draw Thick* box respectively.

More Options . . . The Out of screen adjustment and Transparency Threshold are used when multiple plots are drawn over each other. These options are to prevent the confusion of intersections and other drawing dilemmas that arise from overlapping plots.

Remove Plot Selecting remove plot deletes the plot from the graph window and closes the plot window.

4.4 Surface

Enter parametric surface In the dialogue box, enter the parametric equation of the surface.

Coloring Coloring works as layers. Each plot may have numerous colors layered over each other. Opacity and blending allow the visibility of multiple color schemes concurrently. See *Coloring* section for more details.

Plot is Visible The surface may be hidden by unchecking *Plot is Visible*.

More Options . . . The Out of screen adjustment and Transparency Threshold are used when multiple plots are drawn over each other. These options are to prevent the confusion of intersections and other drawing dilemmas that arise from overlapping plots.

Remove Plot Selecting remove plot deletes the plot from the graph window and closes the plot window.

4.5 Wireframe

Enter parametric surface In the dialogue box, enter the parametric equation of the surface. Surfaces must be dependent on exactly two intervals.

Coloring Coloring works as layers. Each plot may have numerous colors layered over each other. Opacity and blending allow the visibility of multiple color schemes concurrently. See *Coloring* section for more details.

Plot is Visible The surface may be hidden by unchecking *Plot is Visible*.

Draw Thick The thickness of the lines in the wireframe may be increased or decreased by checking or unchecking the *Draw Thick* box respectively.

Connections ... Based on the intervals used to define the parametric expression, by clicking *Connections ...* a small dialogue box will open and list the intervals. Each interval will have a box to the left of the *Connect* text that may be checked or unchecked. By unchecking the box, the wireframe will not connect lines between the specified interval.

More Options ... The Out of screen adjustment and Transparency Threshold are used when multiple plots are drawn over each other. These options are to prevent the confusion of intersections and other drawing dilemmas that arise from overlapping plots.

Remove Plot Selecting remove plot deletes the plot from the graph window and closes the plot window.

4.6 Polygon

Title Enter title for the polygon.

Add Point Successively add points to create the polygon.

Coloring Coloring works as layers. Each plot may have numerous colors layered over each other. Opacity and blending allow the visibility of multiple color schemes concurrently. See *Coloring* section for more details.

Plot is Visible The Curve plot may be hidden by unchecking *Plot is Visible*.

Draw Thick The thickness of the lines in the wireframe may be increased or decreased by checking or unchecking the *Draw Thick* box respectively.

Subdivisions per edge *****need information

More Options ... The Out of screen adjustment and Transparency Threshold are used when multiple plots are drawn over each other. These options are to prevent the confusion of intersections and other drawing dilemmas that arise from overlapping plots.

Remove Plot Selecting remove plot deletes the plot from the graph window and closes the plot window.

4.7 Polyhedron

Title Enter title for the polyhedron.

Add/Remove Vertex Successively add points to create the vertices of the polyhedron.

Add/Remove Face Enter the vertices desired to define each face. Enter in the form: vertex number, vertex number The vertex number is located to the left of the vertex defined in the left-hand column.

Set Face Color . . . Each face may have a different color. The default color for each face is solid green.

Plot is Visible The Curve plot may be hidden by unchecking *Plot is Visible*.

More Options . . . The Out of screen adjustment and Transparency Threshold are used when multiple plots are drawn over each other. These options are to prevent the confusion of intersections and other drawing dilemmas that arise from overlapping plots.

Remove Plot Selecting remove plot deletes the plot from the graph window and closes the plot window.

4.8 Field

Enter the intervals . . . Enter the intervals as a comma-separated list.

Plots Click on *Add Plot* to bring up a drop-down menu of plot options to create the field from. Select the desired plot

5 Coloring

To add or edit the color options for a plot from the plot dialogue box, click *Add Coloring* to begin a new color scheme or click *Edit Coloring* to edit the current color settings. Double-clicking the current color scheme also opens the color editor. The default color scheme is green constant color.

A small color dialogue box will open. In the box chose the desired Coloring Type

5.1 Constant color

Select desired color in Color drop-down menu or make a color by selecting *Other ...* in the drop-down menu. To design own color, select the intensities of red, green and blue and the opacity. The sliders may be moved across the range from zero to 100 or the desired level (from 0 - 100) may be entered in the text box to the right of the color and slider. Decreasing opacity allows the visibility of overlapping color schemes. If there is only one color scheme, decreasing opacity reveals a checkered pattern.

5.2 Expression Spectrum

In the Expression text box enter the desired expression. The interval is often used as the expression entry for the spectrum. The spectrum runs through the colors of the rainbow for every integer step. To create one spectrum (one rainbow) across the entire surface enter `scale (interval)` in the expression text box, eg. `scale (a)`. Note: only intervals and variables can be the arguments of the scale function.

5.3 Gradient

In the Expression text box enter the desired expression. For example, to color by height enter `X_3(a,b)` in the expression text box. The gradient is evaluated and each point is colored by the corresponding color.

5.4 Checkered

Checkered color type is only available for surface, wireframe, and plots. Enter the intervals that define the graph of the function, eg. `a,b`.

Multiple colors may be used for each surface. The layer of colors is created from the top of the list to the bottom. To change the location of the color in the list, drag one color up or down in the list. The opacity and blending options may be used to view colors on top of each other. Blending at 100 or opacity at 50 create the same transparent effect.

Describe Layering!!!!

6 Built-in function and constants

6.1 General functions

The definitions in this file are loaded into the base environment at of the demo program at startup.

Definitions are of the form "name = definition" or "name(args) = definition", in the demo window only the name needs to be typed to complete the desired function. These functions make computations much easier and are available to the user to reduce the time used to enter equations and increase the time available to manipulate graphs and understand the underlying concepts.

Derivative $X_{..1}$ takes the first derivative of X , $X_{..u}$ takes the derivative of X with respect to u .

Integral `integral(integrand, varname, lower, upper [, resolution])`

If `if (condition) then trueexpr else falseexpr` if condition is $\neq 0$, trueexpr is executed, otherwise falseexpr is

Or, And Does boolean or or and, returning either 1 or 0. For the left and right arguments to or and and, "true" is $\neq 0$. (say "a or b")

Equals `a = b` returns 1 iff $\text{abs}(a - b) \leq 1E-10$. Otherwise, 0.

x_min, x_max, x_res returns the min, max, or resolution of an interval or variable

func(argnames)body a function with the argument names argmanes (as a comma-dilinated list), whose body is body. Example: `func(x,y)x+y` is a funciton that returns $x + y$.

Matrix `a,b;c,d` : constructs a matrix with rows a,b and c,d. Can make any number of rows or columns, but it all rows must have the same number of entries.

Scale `Range -j R` a Range is anything with a start, end, and resolution (like an interval or variable) scale linearly maps a range to $[0,1]$ where 0 corresponds to the start of the range, and 1 corresponds to the end
$$\text{scale}(x) = (x - x_{min}) / (x_{max} - x_{min})$$

Identity function `I(x) = x`

Basis2 Given two linearly independent vectors a and b , Basis2 creates the frame $(E1, E2)$, where $E1$ is the unit vector in the direction of a , and $E2$ is the unit vector orthogonal to $E1$ in the direction that b points. In other words, Basis2 does a Gram-Schmidt orthonormalization of (a,b) . Basis2 then returns a function that maps \mathbb{R}^n to \mathbb{R}^2 such a given point x in \mathbb{R}^n is mapped to a point in \mathbb{R}^2 whose first coordinate is $\text{dot}(x,E1)$ and whose second coordinate is $\text{dot}(x,E2)$. In other words, the function maps a point x to its projection in the $E1,E2$ plane, and returns this projection expressed as a point in the $E1,E2$ coordinate system. $\text{Basis2}(a,b) = \text{let } [E1 = \text{unit}(a)] [E2 = \text{unit}(b - \text{dot}(b,E1)*E1)]$
 $\text{func}(x) \text{dot}(x,E1), \text{dot}(x,E2)$

Basis3 Same as Basis2, but makes a basis of 3-space. Can be used to map to 3-space or for redefining the basis. Given three vectors a , b and c , Basis3 returns a function that maps a point x to its projection in the a , b , c space. It expresses this projection in the orthonormal coordinate system obtained from a , b and c . $\text{Basis3}(a,b,c) = \text{let } [E1 = \text{unit}(a)] [E2 = \text{unit}(b - \text{dot}(b,E1)*E1)] [E3 = \text{unit}(c - \text{dot}(c,E2)*E2 - \text{dot}(c,E1)*E1)]$
 $\text{func}(x) \text{dot}(x,E1), \text{dot}(x,E2), \text{dot}(x,E3)$

PlaneToSpace Maps a point in \mathbb{R}^2 to a point in the x - y plane of \mathbb{R}^3 . If given a function that returns a point in \mathbb{R}^2 , PlaneToSpace returns the same function but returns its point in \mathbb{R}^3 . $\text{PlaneToSpace}(x) = (x_1, x_2, 0)$

6.2 Curves

Tangent Vector The Unit Tangent Vector maps a parameterized curve to tangent vector function $\text{TangentVector} = T$
 $T(X) = \text{unit}(X')$

Speed $\text{Speed}(X)(t)$ is $s'(t)$, the speed function of the curve. It is the derivative of arclength.
 $\text{Speed}(X) = \text{length}(X')$

Principal Normal Vector The Unit Principal Normal Vector maps a parameterized curve to principle normal vector function $\text{PrincipalNormalVector} = P$
 $P(X) = \text{unit}(T(X)')$

Curvature Returns the curvature function of a given parameterized curve. Works for $n = 2$ or 3 . $\text{Curvature}(X) = \text{length}(T(X)')/\text{Speed}(X)$

Evolute Evolute maps a parameterized curve to the evolute curve. Works for $n = 2$ or 3 . $\text{EvoluteCurve} = \text{Evolute}$
 $\text{Evolute}(X) = X + P(X)/\text{Curvature}(X)$

Osculating Circle Maps a parameterized plane or space curve at a given point to its osculating circle at that point. The point is specified as a value for the parameter of the curve. The osculating circle is given as a circle function: it takes a parameter between 0 and 2π .
 $\text{OscCircle}(X,t) = \text{func}(\text{angle})(X(t) + (P(X)(t) + \cos(\text{angle}) * T(X)(t) + \sin(\text{angle}) * P(X)(t)) / \text{Curvature}(X)(t))$

6.3 Plane Curves

Normal Plane Curve Normal vector to curve: maps parameterized curve to unit normal vector function U is T rotated $\pi/2$ counter-clockwise
 $\text{PlaneCurveNormal} = U$
 $U(X) = (-T_2(X), T_1(X))$

6.4 Differential Geometry Functions - Space Curves

Unit Binormal Vector The Unit Binormal Vector: maps parameterized curve in three-space to binormal vector function. $\text{BinormalVector} = B$
 $B(X) = \text{unit}(\text{cross}(X', X''))$

Torsion Torsion maps a parameterized space curve to its (parameterized) torsion.
 $\text{Torsion}(X) = \text{dot}(\text{unit}(\text{cross}(X', X'')), X''')$

6.5 Surfaces

Unit Normal Vector for Surface The Unit Normal Vector: maps parameterized surface in three-space to unit normal function $\text{NormalVector} = N$
 $N(X) = \text{unit}(\text{cross}(X_{..1}, X_{..2}))$

Metric Coefficients Metric coefficients: Given a parameterized surface, g_{11} , g_{12} , g_{21} , and g_{22} return the metric coefficient for the surface as a function from the domain to \mathbb{R} .
 $\text{MetricCoef11} = g_{11}$
 $\text{MetricCoef12} = g_{12}$
 $\text{MetricCoef21} = g_{21}$

$\text{MetricCoef22} = g_{22}$
 $g_{11}(X) = \text{dot}(X_{_1}, X_{_1})$
 $g_{12}(X) = \text{dot}(X_{_1}, X_{_2})$
 $g_{21}(X) = \text{dot}(X_{_2}, X_{_1})$
 $g_{22}(X) = \text{dot}(X_{_2}, X_{_2})$

Matrix of metric coefficients $\text{MetricCoefMatrix} = \text{gmatrix}$
 $\text{gmatrix} = [g_{11}, g_{12}; g_{21}, g_{22}]$

L coefficients $L_{11}, L_{12}, L_{21}, L_{22}$:

$L_{11}(X) = N(X) * X_{_1_1}$
 $L_{12}(X) = N(X) * X_{_1_2}$
 $L_{21}(X) = N(X) * X_{_2_1}$
 $L_{22}(X) = N(X) * X_{_2_2}$

TotalCurvature Total Curvature / Gaussian Curvature: maps a parameterized surface to its total curvature function. $\text{GaussianCurvature} = \text{TotalCurvature}$

$\text{TotalCurvature}(X) = \text{dot}(\text{cross}(N(X)_{_1}, N(X)_{_2}), \text{cross}(X_{_1}, X_{_2})) / \text{length}(\text{cross}(X_{_1}, X_{_2}))^2$

Hessian determinant The Hessian determinant maps a parameterized surface to its Hessian.

$\text{Hessian}(X) = 0.5 * ((L_{11}(X) * g_{11}(X) - 2 * L_{12}(X) * g_{12}(X) + L_{22}(X) * g_{22}(X)) / (g_{11}(X) * g_{22}(X) + g_{12}(X)^2))$

6.6 Common Shapes

Circle Given a radius R , returns a circle of radius R in the plane centered at the origin. The circle is a function of an angle parameter in the range $[0, 2 * \pi]$

$\text{Circle}(R) = \text{func}(\text{angle}) R * (\cos(\text{angle}), \sin(\text{angle}))$

Disc Given a radius R , returns a disc of radius R in the plane centered at the origin. The disc is a function of two parameters (radius, angle). To fill the disc, angle should be in $[0, 2 * \pi]$, and radius should be in $[0, 1]$.

$\text{Disc}(R) = \text{func}(\text{radius}, \text{angle}) R * \text{radius} * (\cos(\text{angle}), \sin(\text{angle}))$

Sphere Given a radius R , returns a sphere of radius R in 3-space centered at the origin. The sphere is a function of two parameters (θ , ϕ). To cover the sphere exactly once, θ should be in $[0, 2 * \pi]$ and ϕ should be in $[0, \pi]$.

$\text{Sphere}(R) = \text{func}(\theta, \phi) R * (\cos(\theta) * \cos(\phi), \cos(\theta) * \sin(\phi), \sin(\theta))$

Normal Plane Given a vector n , returns the plane normal to n through the origin. The plane is given as a function of two parameters (u,v) . These parameters are the variables for an orthonormal coordinate system in the plane.

PlaneNormal(n) = let [a = if (unit($n + (1,0,0)$)*length(n)/2) = unit(n)) then ($n + (0,1,0)$) else ($n + (1,0,0)$)*length(n)/2)] [E1 = unit(cross(n , a))] [E2 = unit(cross(E1, n))] func(u,v) u *E1 + v *E2

Tube Given a parameterized space curve and a radius, returns the tube around the curve. The tube is given as a function of two parameters (t, θ) . To cover the tube exactly once, θ should be in $[0, 2\pi]$. t is the parameter for the curve.

Tube(X, R) = func(t, θ) $X(t) + R$ *cos(θ)*P(X)(t) + R *sin(θ)*B(X)(t)

Osculating Tube Given a parameterized space curve, returns the osculating tube of the curve. The tube is given as a function of two parameters (t, θ) . To cover the tube exactly once, θ should be in $[0, 2\pi]$. t is the parameter for the curve.

OscTube(X) = func(t, θ) $X(t) + (\cos(\theta)*T(X)(t) + (1+\sin(\theta))*P(X)(t))/Curvature(X)$

Cylinder Given a plane curve and a vector, returns the cylinder created by pushing the curve (lying in the x-y plane) along the given vector. The cylinder is given as a function of two parameters (t,v) , where v is the distance along the vector and t is the parameter for the curve.

Cylinder(X, A) = func(t,v) ($X_1, X_2, 0$)(t) + v *A

Tangent Strip Given a space curve, returns the tangential strip of the curve. The tangential strip is given as a function of two parameters, (t,v) . t is the parameter for the curve, and v is the distance along the tangent vector to go.

TangStrip(X) = func(t,v) $X(t) + v$ *T(X)(t)

Surface of Revolution Given a plane curve, returns the surface of revolution of the curve. The curve is treated to be in the x-y plane, and is revolved about the y-axis. The surface is given as a function of two parameters (t, θ) . t is the parameter for the curve, and θ is the angle of the revolution. To cover the surface exactly once, θ should be in $[0, 2\pi]$.

RevSurface(X) = func(t, θ) cos(θ)* $X_1(t), X_2(t), \sin(\theta)$ * $X_1(t)$

7 Hotspots

To show the location of the Hotspot in the 3D Graph window, from the Plot drop-down menu in the 3D Graph window select "Add Point..." Enter $X(A)$ or desired function (A) in the "Enter a point" text box and press the enter key. When the hotspot is moved in the 2D graph, the image of the point will also move in the 3D Graph.

8 Making a simple demo from scratch

1. Open a blank demo by selecting *New Demo* from the Controls Window Demo drop-down menu. The new Controls window will begin as a blank window.
2. Input a function by selecting *Add New Function* in the Controls Window Controls drop-down menu. Intervals for the domain of the function must be added before the function may be graphed
3. Add intervals by selecting "Add New Interval" in the Controls Window Controls drop-down menu.

9 Graphing a function

Inserting a 2D Graph:

To insert a 2D Graph select "New 2D Graph" from the Controls drop-down menu in the Controls window. A blank 2D Graph with the x and y axis will appear.

To add a wireframe, which is a set of line segments joining points on the graph, from the plot drop-down menu select "Add Wireframe...".

In the "Enter a parametric expression" text box enter the expression with the desired intervals. For a rectangular domain, enter the intervals, e.g. **a**, **b**.

By changing the number of steps for each interval in the Controls window, the number of line segments increases and decreases correspondingly.

The "Connections" selection in the Wireframe edit window allows for the removal or addition of lines between points of the interval. If all connections are removed, points remain at the locations of the intersection of the invisible lines.

10 3D graph

To graph a 3D function in the Controls drop-down menu select *New 3D Graph*

The x, y, and z-axis will be visible in the 3D Graph window

From the Plot drop-down menu, select *Add Plot* and select *Surface*, in the text box below the text "Enter a parametric surface" type the desired function to be graphed. If the function has been entered into the Controls window, type the form of the function and the intervals, eg. $X(a,b)$.

INSERT SIMPLE DEMO HERE

11 Editing a function in the 3D Graph view

To edit, change the coloration or remove a function from the 3D Graph window, in the Plot drop-down menu there will be a list of the surfaces at the bottom of the menu. Select the desired surface.

To keep the function in the plot drop-down menu without displaying the surface on the 3D graph, uncheck *Plot is Visible*. To redisplay the surface, recheck *Plot is Visible*.

To remove the surface completely, select *Remove Plot*.