Secrets of My Success

Thomas F. Banchoff

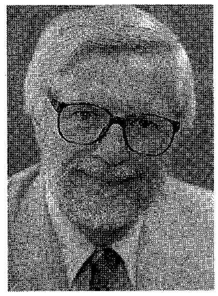
Remarks on receiving the MAA's 1995 Deborah and Franklin Tepper Haimo Award for Distinguished College or University Teaching of Mathematics, Orlando, Florida, January 1996.

There really isn't much secret to what counts as success in teaching. You are teaching well when your students are involved, working hard, and learning worthwhile things that can help them throughout their lives. So what can a teacher do to foster that?

In my experience, students will get involved if they see that the teacher is involved. They will work hard if the teacher does. And they will believe that what they are learning is worthwhile if the teacher shows that it is worthwhile in his or her own life and in the lives of former students.

Giving a brief talk about a thirty-year career is a challenge, and perhaps the best way is to present a scrapbook of anecdotes and images (even though the written version will miss out on the slides and videotapes). As I prepare for this, it becomes clear how much my teaching has changed and continues to change. We are always telling our students that we can't teach them the mathematics they will need in the next century, but we can only teach them how to keep learning. Until recently I had not realized how much that advice also applies to us as teachers.

To illustrate the great changes, I wish I could immediately switch to an interactive presentation on the Internet. Within a very few years, every convention center will be outfitted with such technology, and anywhere in the world it will be possible to have access to resources from everywhere else. What a change that will make in the audio-visual scene. Throughout my career, I have collaborated with computer science colleagues and students to produce graphics for my courses, my research, and my lectures. Three decades ago, it was hard enough to locate a working 16 mm projector, or a tray that would fit all slide projectors. For a few years, in order to show a videotape, you had to bring with you the equipment it was recorded on, but then



almost overnight the VHS standard became universal. Technology continues to improve and to become more accessible at incredible rates, and the Internet promises to accelerate that process. It is already changing my teaching and my lecturing in ways I could not have anticipated.

But that is the future, and I promised some scrapbook items from the past. Now I routinely teach college students one-third my age, but when I started, I had one memorable student three times as old as I was. Sr. Adelicia was in an NSF summer biology program at Notre Dame in 1960 when I stayed on after graduation to be an assistant for Dr. Arnold Ross in his program for high school teachers and very bright high school students. My assignment was a recitation section for assorted students from other departments trying to get a required math credit by taking elementary number theory. Most of my students had a terrible time with the first exam, and I called them in one at a time to go over the results. Sr. Adelicia had received a score of 20, and when she arrived, before she had a chance to say anything, I showed her how she almost had the right answer in a couple of problems and how she could organize things to make headway on some of the others—I was sure she could improve. "Do you really think so?" she asked and she went on her way.

When the next exam grades came out, everyone had improved, but no one as much as Sr. Adelicia, up to a low passing grade. Everyone in the section applauded when I announced that she had more than doubled her previous score, and she was as proud as could be. She came to my office and said, "I did it for you."

"What?" I asked.

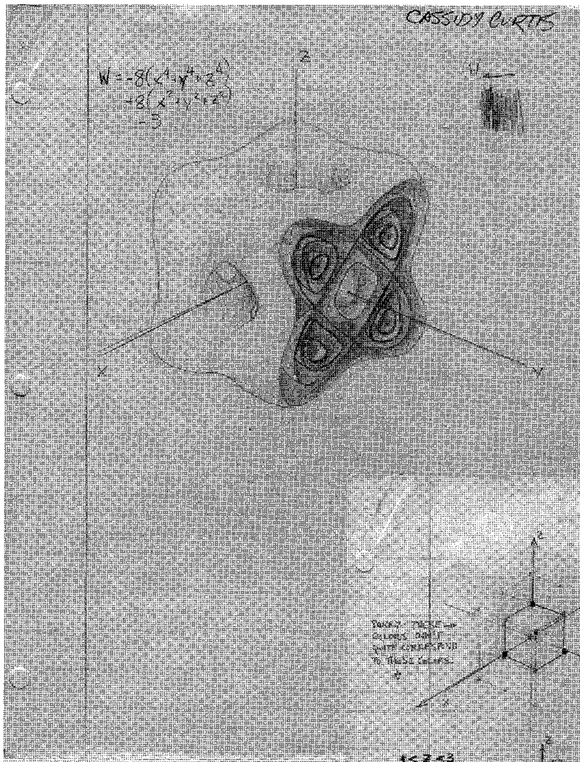
She continued, "When I came to your office after the first test, it wasn't to ask for help. I had decided to drop out of the whole program and go back home, but you had faith in me. You gave me the confidence to give it one more try. I did it for you."

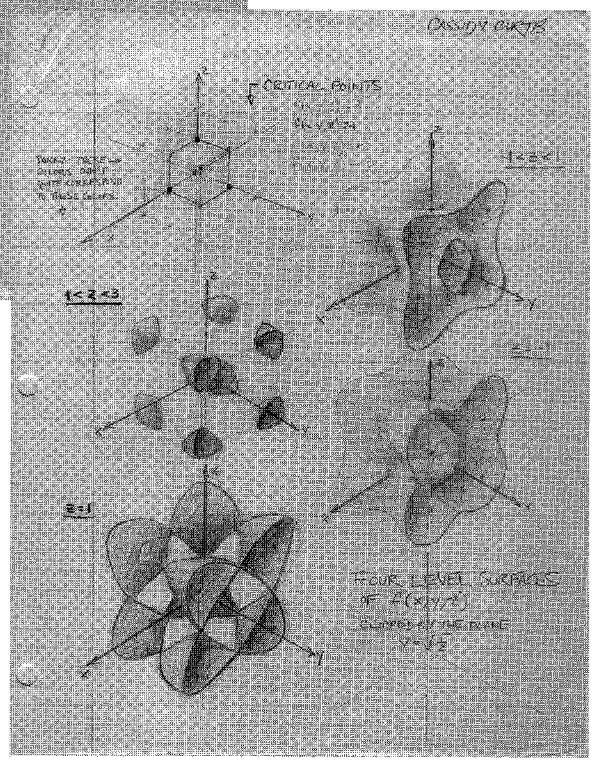
I was flabbergasted. This was my first experience teaching and I was getting the kind of affirmation I thought only came at the end of a long career. I seriously wondered if it was all going to be downhill from then on. Maybe I should quit while on top.

The next anecdote is about the worst course evaluation I ever received from a student. I wish I could say it came when I was naive and inexperienced, but it was this past semester. Almost all of the students in my differential geometry course appreciated it very much, but one was unhappy nearly all of the time. Everything that worked for the others turned him off. I emphasized group projects, and he only wanted to work alone. The class delighted in coming up with conjectures, writing up their ideas each day, and then discussing them when I handed them back with comments the following class; but the kind of homework he wanted was lists of problems from the book, assigned several weeks in advance. Open-ended problems on take-home exams? No thanks. Interactive visualizations in the weekly computer laboratories? It might work for other people, but it didn't do anything for him. Never before had I seen an evaluation with the lowest possible rating in every category, except for his own effort.

What do you do about that kind of outlier? You just have to accept the fact that you can't please everybody, especially if you want to be open to trying new things. As a teacher, you have to strive to challenge almost everyone in the class, trying to keep the quickest ones from getting bored and the slowest from getting lost. But you can't abandon the whole flock to chase after one who doesn't want to go along. You have to remember that good teaching is not identical with perfect ratings.

Assessment is a constant theme for teachers, and I want to share a device that works for me: the Two-Phase Hour Exam. It came about many years ago thanks to the inventiveness of one freshman student, Nick Nickerson. I gave a midterm in the calculus class on the day before Thanksgiving, and, with his suitcase waiting for him in the back of the room, he wrote a miserable exam. I could see how frustrated he looked as he rushed off for his train. As I graded his test, I could see why—his errors and omissions made it quite difficult to determine whether he knew anything at all. But the day after Thanksgiving, I received in the mail a packet from Nick saying that he had sat down immediately in the train and in one hour he had written answers to all of the exam questions, perfectly as it turns out. He told me he had studied very hard and he knew the material, but he just got flustered at the beginning of the exam and he never got back on track. He wasn't trying to make an excuse, he said, and he had no idea what I might want to do with his second effort, but he did want me to know that that hour exam did not represent how well he knew the material in the course.





Entries for the "Best Homework Ever"

I thought quite a bit about Nick's letter. I realized that I had learned two things: how well Nick could do in a pressure situation, and how well he could do when he had more time to think carefully. Both pieces of information are important.

From that time onward, whenever I give a timed exam in a course, I instruct everyone to do what Nick did instinctively. Phase I: After one hour of an in-class exam, they hand in their exam book and it will be graded before the next class. Phase II: Take the exam questions home and do the same test carefully and completely, and hand it in at the next class period. On the second phase, students can use notes and books and any kind of computer, but they are not to discuss the test with anyone.

I did not originally anticipate all the effects of this procedure, and I am convinced that it is beneficial in quite a few ways. For one thing, I no longer get students crowding around the desk at the end of the exam complaining that it was too long, and that they could have done so much better with just a little more time. They have an opportunity to show me what they can do on the next phase.

And they do show me. It is easy to evaluate the second part because just about everyone does very well, but some students will do very well indeed, coming up with elegant solutions and catching the subtleties that are easy to miss in the hour exam pressure. It is also clear which students are still fundamentally confused, and both phases can form the basis for a discussion of the difficulties. Even when there is widespread confusion, the subsequent class discussion can focus on the difficulty productively when everyone has been thinking and writing about it. In most cases, after such an exam, just about everyone understands what is important up to that point and we are ready to go on to the next topic.

Invariably students ask how I "count" the two phases, and I always answer that I will take both into consideration in making up my evaluation. I am much more interested in where they end up in the course than in their showing in the preliminaries. Phase I identifies the mathematical sprinters and Phase II, the middle distance runners. Both aspects are important, and we ignore a great deal of mathematical talent if we only look at the one measure of performance. I wonder how many potential deep-thinking mathematicians were weeded out in the time trials of yesteryear?

Students almost always consider my exams challenging, since I try to design them so that only one or two of them, working very quickly and accurately, will be able to do the in-class part perfectly in an hour. I am very disappointed if no one does the test very well, and equally disappointed if anyone finishes early. I always put in one or two extra credit problems so no one gets bored, and for some students, the chance to work on these more interesting problems during the second phase is the most satisfying part of the course. After two or three of these tests, students begin to catch on and performance on both phases improves. On the three-hour final, equivalent in length to about two hour exams, people do have time to finish—there isn't any Phase II in the Last Judgment.

An example of a benefit from the Two-Phase experience occurred this past semester in the midterm of my honors multivariable calculus course. I had asked the students to find the centroid of a region bounded by a quadrilateral symmetric to the x-axis, and in the hour test, even some of the best students jumped to the (incorrect) conclusion that the centroid would always be the vector average of the vertices. On Phase II, a number of students recognized that the problem was more subtle and came up with alternate strategies for solving it. In subsequent class discussion we formulated conjectures about the relationship between the centroid and the average, and we identified those quadrilaterals for which the centroid was outside the region. Two of those students who contributed most to this discussion will be working with me this summer to adapt our multivariable calculus interactive laboratory materials for use in my introductory calculus sequence this fall.

The pattern from this past semester repeats an experience I have enjoyed for many years—students in an elementary class become turned on to mathematical research, they follow up their interest in independent studies or summer internships, and they become co-workers in developing new pedagogies, especially using computer graphics and hypertext systems. I am very proud of the student assistants who have gone on to graduate school and to positions in teaching, publishing, computer animation, and design.

One more scrapbook item that does demand an illustration even in a printed version is the Best Homework Ever. Every teacher probably has a candidate for this award, and I think that my entry will stand up against any of them. I always ask my students to learn to draw, especially in the two-variable calculus course where so much of the intuition comes from exploring representations of function graphs in three-dimensional space. I have had some fine artists in my classes, none so impressive as Cassidy Curtis. Ten years ago, as a freshman in my third-semester calculus course, he excelled from the beginning, choosing just the right viewpoint, shading, and coloring for rendering surfaces in three-space. He was equally good with colored pencils and with computer graphics.

After the students had worked a good deal with contour lines of functions of two variables, I introduced the analogous concept for three variables and challenged them to investigate contour surfaces.

I had a particular example in mind. The previous summer, in a lecture series on complex algebraic surfaces at the Mathematical Sciences Research Institute in Berkeley, Professor Friedrich Hirzebruch had described a polynomial in three complex variables

$$f(x, y, z) = (8x^4 - 8x^2 + 1) + (8y^4 - 8y^2 + 1) + (8z^4 - 8z^2 + 1).$$

One level set had a large number of singular points, and although he talked about its algebraic and analytic properties, he regretted he did not have any pictures to show the geometry of the surface. I phoned back to Brown and one of my undergraduate assistants used our implicit function renderer (a senior project of the previous year) to produce some slides of the real part of the surface. Professor Hirzebruch was delighted to use them in his next lecture three days later.

When I introduced contour surfaces to Cassidy's class, I had planned to lead up to this story after a couple of weeks of preparation, but I made the mistake of writing the equation on the board without telling the class how difficult the problem was. The very next class, two days later, Cassidy Curtis said, "I figured out what your surface looks like. I decided to show a whole set of levels, sitting inside a cube, and I removed one face of the cube so it is easier to see the structure." I was astounded to see a perfect image of the surface we had rendered with such labor. I was even more impressed when he showed the next page where he had stacked all the color-coded surfaces together, something our computer could not do at the time!

I have shown these images in many, many lectures. I claim that not only is this the best overnight homework I have ever received from a freshman—this is the best homework that anybody has ever received in any course at any level in any subject at any place, ever ever ever. That may be an exaggeration, but I am still waiting for someone to produce a counterexample.

One final anecdote is especially appropriate for this award presentation. I remember many of the students from the first calculus course I ever taught, as a Benjamin Peirce Instructor at Harvard in 1964, and I was pleased that two of them wrote supporting letters when I was considered for this award. One was Zara Haimo. I knew her parents before they were my colleagues, so I am so very pleased to receive this award named in honor of Debbie and Frank Haimo.

So many teachers and so many students have influenced me over the years. Whatever success I have had comes from working with them, all trying to do our best. What lies ahead to challenge us as teachers? I don't know, but I'm looking forward to it.

Tom Banchoff teaches at Brown University in Providence, Rhode Island. His e-mail address is tfb@cs.brown.edu.