

LOGARITHMIC EVALUATION SPACES

DAN ABRAMOVICH

All log schemes here should probably be fine and saturated.

1. LOG POINTS AND THEIR MAPS

We want to construct the stack $\wedge X$. We start with $\wedge' X$. This is a log space whose points over a log scheme S are maps $S \times p^\dagger \rightarrow X$. The automorphism group \mathbb{G}_m of p^\dagger acts on $\wedge' X$. We have

Lemma 1.1. $\wedge X = [\wedge' X / \mathbb{G}_m]$.

We quote from [1]:

Lemma 1.2. *If*

$$X = \varprojlim X_i$$

then

$$\wedge X' = \varprojlim \wedge' X_i.$$

Problem 1.3. There is an analogous result for $\wedge X$ - state and prove it!

Also we have

Lemma 1.4. *If X carries a trivial log structure then $\wedge' X = X$.*

Finally we have

Problem 1.5. Let $U_i \rightarrow X$ be a strict étale cover. Write U_{ij} for $U_i \times_X U_j$ as usual, so

$$X = \varinjlim (\{U_{ij}\} \rightrightarrows \{U_i\}).$$

Show that

$$\wedge' X = \varinjlim (\{\wedge' U_{ij}\} \rightrightarrows \{\wedge' U_i\}).$$

Show the same for $\wedge X$.

Lemma 1.6 (Gillam). *Let*

$$X = \text{Spec}(\mathbb{N} \rightarrow \mathbb{Z}[\mathbb{N}]).$$

Then

$$\wedge' X = \sqcup_{i \in \mathbb{N}} \text{Spec}(\mathbb{N} \rightarrow \mathbb{Z}[\mathbb{N}]),$$

the action of \mathbb{G}_m on the i -th component is through the character $t \mapsto t^i$, the 0 component maps to X isomorphically, and all other components map to the origin.

Date: today.

Problem 1.7. Construct $\wedge'X$ and $\wedge X$ for all fine and saturated X as follows:

- (1) X has an étale covering by charts U_i of the log structure. By a previous problem it is enough to construct $\wedge'U_i, \wedge'U_{ij}$. So we may assume X has a global chart.
- (2) If X has a chart with monoid P we can write

$$X = \underline{X} \times_{\mathrm{Spec} \mathbb{Z}[P]} \mathrm{Spec}(P \rightarrow \mathbb{Z}[P]).$$

By a previous problem it suffices to treat

$$X = \mathrm{Spec}(P \rightarrow \mathbb{Z}[P]).$$

- (3) Choose P a presentation

$$\mathbb{N}^k \rightrightarrows \mathbb{N}^l \rightarrow P$$

then X is an equalizer

$$X \rightarrow (\mathrm{Spec}(\mathbb{N} \rightarrow \mathbb{Z}[\mathbb{N}]^l) \rightrightarrows (\mathrm{Spec}(\mathbb{N} \rightarrow \mathbb{Z}[\mathbb{N}]^k).$$

By a previous problem we have $\wedge'X$ is an equalizer

$$\wedge'X \rightarrow (\wedge' \mathrm{Spec}(\mathbb{N} \rightarrow \mathbb{Z}[\mathbb{N}]^l) \rightrightarrows (\wedge' \mathrm{Spec}(\mathbb{N} \rightarrow \mathbb{Z}[\mathbb{N}]^k),$$

and each term is computed by Gillam's lemma.

Problem 1.8. Repeat the same for other types of log points, eg log nodes.

REFERENCES

Abramovich-Chen
Kresch
Manolache

- [1] <http://www.math.brown.edu/~abrmovic/PAPERS/LOG/logstab.pdf>
- [2] Kresch's thesis.
- [3] Manolache's thesis.