EXAMPLES AND EXERCISES FOR: LOGARITHMIC GEOMETRY AND MODULI LECTURES AT THE SOPHUS LIE CENTER JUNE 16-17, 2014

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(1) Is $(\underline{X}, \mathcal{O}^*_{\underline{X}} \hookrightarrow \mathcal{O}_{\underline{X}})$ the trivial structure, coherent? Inegral? Saturated?
(2) what about $(\underline{X}, \mathcal{O}_{\underline{X}} \stackrel{\sim}{\to} \mathcal{O}_{\underline{X}})$?
(3) Calculate explicitly the logarithmic structure associated to$(\underline{X}, \mathbb{N} \stackrel{\alpha}{\to} \mathcal{O}_{\underline{X}})$, where α is determined by a choice of $\alpha(1)$.
(4) Consider the divisor $D = \{z = 0\}$ in the threefold $xy = zw$. Is the associated logarithmic structure coherent?
(5) Find the automorphism group of the standard logarithmic point.
(6) Let X and Y be standard logarithmic point and Z an $\mathbb{N}^2$- logarithmic point. Let $f : X \to Z$ be given by sending $(1, n) \mapsto (1, n, n)$ and $g : Y \to Z$ be given by $(1, n) \mapsto (z^n, an, bn)$. Calculate the fibered product $X \times_Z Y$ when
   - $z = a = b = 1$
   - $a = b = 1, z \in k^\times$ arbitrary,
   - in general.

   Under what conditions is the result integral?
(7) Calculate the sheaf Ω^1_X when X is a toric variety with its standard logarithmic structure.
(8) Calculate the sheaf Ω^1_X when X is a p-logarithmic point, for a toric monoid P.
(9) calculate the infinitesimal automorphisms of $\mathbb{P}^1$ as a toric logarithmic scheme.
(10) Show explicitly that the family of curves $xy = t$ is logarithmically smooth at the node.
(11) Show explicitly using the characterization that the normalization $\operatorname{Spec}(\mathbb{N} \to \mathbb{C}[\mathbb{N}]) \to \operatorname{Spec}(\mathbb{N}_{\smallsetminus 1} \to \mathbb{C}[\mathbb{N}_{\smallsetminus 1}])$ is not integral
(12) do the same for the blowup $\operatorname{Spec} \mathbb{C}[x, y] \to \operatorname{Spec} \mathbb{C}[x, z]$ given by $z = xy$.

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(13) Show that the three definitions of stable curves are equivalent.
(14) What are all the possible logarithmically smooth structures on the nodal family $xy = t$?

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