Dan, 

Yes, I agree that the functor \( \text{LogSch}^{op} \to \text{Sets} \)
\[ X^\dagger \mapsto \text{Hom}_{\text{LogSch}}(X^\dagger \times k^\dagger, k^\dagger) \]
(here \( k^\dagger := \text{Spec}(0 : \mathbb{N} \to k) \)) and the product is the one in log schemes over \( k \) is represented by
\[ \text{Spec} k \coprod \coprod_{d > 0} \text{Spec}(x : \mathbb{N} \to k[x]). \]

Indeed, if \( X^\dagger = (X, \mathcal{M}_X) \), then \( \text{Hom}_{\text{LogSch}}(X^\dagger \times k^\dagger, k^\dagger) \) is just the set of commutative diagrams

\[
\begin{align*}
\mathcal{M}_X(X) \oplus \mathbb{N}(X) & \xrightarrow{(\alpha, 0)} \Gamma(X, \mathcal{O}_X) \\
\mathbb{N} & \xrightarrow{0} k \\
\end{align*}
\]

and this certainly splits as a disjoint union over the possible \( d := \pi_2 h(1) \in \mathbb{N}(X) \). When \( d = 0 \), commutativity forces \( m := \pi_1 h(1) \in \mathcal{M}_X(X) \) to be such that \( \alpha(m) = 0 \in \Gamma(X, \mathcal{O}_X) \) and the map is the pullback of \( \text{Id} \in \text{Hom}_{\text{LogSch}}(k \times k^\dagger, k^\dagger) \). After passing to a component of \( X \) where \( d \in \mathbb{N}_{> 0} \) is constant, the map in question fits into a diagram

\[
\begin{align*}
\mathbb{N} \oplus \mathbb{N} & \xrightarrow{(x, 0)} k[x] \\
\mathcal{M}_X(X) \oplus \mathbb{N} & \xrightarrow{(\alpha, 0)} \Gamma(X, \mathcal{O}_X) \\
\mathbb{N} & \xrightarrow{h=(m,d)} \Gamma(X, \mathcal{O}_X) \\
\end{align*}
\]

so is pulled back from the universal map given by \( (\text{Spec}(x \to x) \text{ of}) \) the “big” square.

–Danny