Quizzes given in Math 10, Fall 2005

Time for quiz is \( n \) minutes, where \( n \to +\infty \)

Quizzes from Section 1

Quiz for week 10

Determine whether the following series converge or diverge.

Problem 1.

\[
\sum_{n=1}^{+\infty} (-1)^n \frac{\sqrt{n}}{1 + 2\sqrt{n}}
\]

Problem 2.

\[
\sum_{n=1}^{+\infty} (-1)^{n+1} \frac{n^2}{n^3 + 4}
\]

Problem 3.

\[
\sum_{n=0}^{+\infty} \frac{1 + \sin n}{10^n}
\]

Quizzes from Section 2

Quiz for week 1

Problem 1. Evaluate

\[
\int x^2 \cos(x^3 + 1) \, dx
\]

Problem 2. Compute

\[
\int_{-\pi/2}^{\pi/2} \sin(x) \cdot \ln(\cos^2(x) + 2) \, dx
\]

Quiz for week 2

Problem 1. Compute the volume of the solid obtained by rotating the region bounded by \( x = y^2 - 2y \) and \( x = 0 \) about the \( x \)-axis.

Problem 2. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the first region bounded by \( y = 1 - \cos(\frac{\pi x}{2}) \) and \( y = \sqrt{x} \) about the line \( y = 2 \).

Quiz for week 3

Problem 1. Find \( \int e^{3x} \, dx \).

Problem 2. Compute

\[
\frac{d}{dx} \int_{\sin(x)}^{4} e^t \, dt.
\]
Quizzes given in Math 10, Fall 2005

Quiz for week 4

Problem 1. Compute \( \int \frac{dx}{\sqrt{x^2 + 16}} \).

Problem 2. Compute \( \int \frac{x^2 + 2x + 2}{(x^2 + 4)(x - 1)} \, dx \).

Quiz for week 10

Determine whether the following series converges or not. Justify your answer. You can use any of the methods that we’ve discussed, but there is a way to solve these without using ratio/root test.

Problem 1. \( \sum_{n=1}^{\infty} \frac{\sqrt{n^3 + 4 + n}}{n^2 + (n^8 + n^6 + n)^{1/3}} \)

Problem 2. \( \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln n}{\sqrt{n}} \)

Quizzes from Section 3

Quiz for week 1

Problem 1. Fill in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>( \frac{\pi}{4} )</th>
<th>( \frac{\pi}{2} )</th>
<th>( \pi )</th>
<th>( \frac{3\pi}{2} )</th>
<th>2( \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin x )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \cos x )</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( \tan x )</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Quiz for week 2

Problem 1. Find the area of the region bounded by \( y = \sin x \), \( y = 0 \).

Problem 2. Revolve the region from Problem 1 around \( y = -1 \) and find the volume of the solid that is obtained. [Hint: \( \sin^2 x = \frac{1}{2}x - \frac{1}{4}\sin 2x \).]

Quiz for week 3

Problem 1. Evaluate

\[
\int_{0}^{\pi/2} \sin^2 x \cos^2 x \, dx.
\]
Quiz for week 9

Problem 1. Write an expression for the partial sum $s_n$ of the series $\sum_{n=1}^{+\infty} a_n$ where

$$a_n = \frac{1}{n^2} - \frac{1}{(n+1)^2}.$$ 

Quiz for week 10

Problem 1. Use the integral test to determine whether the following series is convergent or divergent

$$\sum_{n=1}^{+\infty} \frac{1}{n^4}.$$ 

Quizzes from Section 4

Quiz for week 1

Problem 1. Use the substitution rule to evaluate the integral

$$\int x^2 \sqrt{1 + x^3} \, dx.$$ 

Problem 2. Use symmetry to evaluate the integral

$$\int_{-2}^{2} \sin x^3 \, dx.$$ 

[Hint: Draw a rough sketch of the function $\sin x^3$.]

Problem 3. Find the area between the curves $y = 2 - x^2$ and $y = 1$.

Quiz for week 2

Problem 1. Find the volume of the solid obtained by rotating the region bounded by the curve $y = \frac{1}{x}$ and the lines $x = 1$, $x = 2$, around the $x$-axis.

Problem 2. Find the average value of the function $f(x) = 2x \cos(x^2)$ on the interval $[\pi, 2\pi]$.

Problem 3. Use integration by parts to evaluate the integral

$$\int x \cos x \, dx.$$
Quiz for week 3

Problem 1. Define

\[ g(x) = \int_{x^2}^{\sin x} e^t \, dt. \]

Find the derivative \( g'(x) \).

Take-Home Problem for week 3 [For people making up Quiz 1]

Problem 1 (This is Exercise 84, p. 422). Use the substitution \( u = \pi - x \) to show that

\[ \int_0^\pi x f(\sin x) \, dx = \frac{\pi}{2} \int_0^\pi f(x) \, dx. \]

Quiz for week 4

Problem 1. Give the partial fraction expansion of

\[ \frac{x^2 + 1}{x^3 + x^2 + x}. \]

Problem 2. Complete the square \( x^2 + x + 1 \).

Problem 3. Evaluate

\[ \int \frac{(\tan^2 \theta + 1) \sec^2 \theta}{\tan^3 \theta + \tan^2 \theta + \tan \theta} \, d\theta \]

Quiz for week 6 [Not given]

Problem 1. Find the circumference of the circle \( x^2 + y^2 = 1 \) by using arclength.

Quiz for week 7

Problem 1. Find what direction field belongs to the differential equation

\[ \frac{dy}{dx} = y^2 + y + \frac{1}{2}. \]
Problem 2. Solve the differential equation in Problem 1.

Problem 3. Find the solution that satisfies \( y(0) = \frac{1}{2} \) and sketch it on the direction field you determined in Problem 1.

Quiz for week 10

Problem 1. Evaluate the infinite sum: \( 1 + \frac{1}{e} + \frac{1}{e^2} + \cdots \).

Problem 2. Let \( b_n = \frac{n}{e^n} \) and show that \( \lim_{n \to +\infty} b_n = 0 \).

Problem 3. Consider the sequence of partial sums \( s_n = b_1 + \cdots + b_n \) (with \( b_n \) as above) and show that
\[
e \cdot s_{n+1} - s_n = 1 + \frac{1}{e} + \cdots + \frac{1}{e^n}.
\]

Quiz for week 10 [For students who took the make-up for Midterm2]

A sequence \( \{a_n\}_{n=1}^\infty \) is defined by
\[
a_n = \frac{1}{n^2} - \frac{1}{(n+1)^2}.
\]
Problem 1. Find \( \lim_{n \to \infty} a_n \).

Problem 2. Write an expression for the partial sum \( s_n \) of the series \( \sum_{n=1}^{\infty} a_n \).

Problem 3. Show that the sequence of partial sums \( \{s_n\}_{n=1}^{\infty} \) converges and find the sum \( \sum_{n=1}^{\infty} a_n \).