Problem 2. Consider the solid formed by rotating the region $0 \leq y \leq e^{x^2}$, with $0 \leq x \leq 2$ about $4y$-axis.

(a): Set up an integral that represents the volume of this solid.

Solution: We use shell method: the radius here is x, the height is $y(x) = e^{x^2}$ and x is from 0 to 2. Hence the volume is

$$Volume = \int_0^2 2 \cdot \pi \cdot x \cdot e^{x^2} dx.$$

(b) Evaluate the integral you set up in part (a).

Solution: Taking a substitution $u = x^2$, then $du = 2x \cdot dx$, plug into the answer of part (a),

$$\int 2 \cdot \pi \cdot x \cdot e^{x^2} dx = \int \pi e^u du = \pi \times e^u + C = \pi \times e^{x^2} + C.$$

Hence,

$$Volume = \int_0^2 2 \cdot \pi \cdot x \cdot e^{x^2} dx = (\pi \times e^{x^2} + C)|_0^2 = \pi \cdot e^4 - \pi \cdot e^0 = \pi \cdot (e^4 - 1).$$

Problem 4.

(a) Compute the value of

$$\int_0^{\frac{\pi}{4}} xsin(x)dx.$$

Solution: Let $u = x, dv = sin(x)dx$, then we have $du = dx, v = -cos(x)$. Using integration by part formula,

$$\int xsin(x)dx = x \cdot (-cos(x)) - \int (-cos(x))dx = -x \cdot cos(x) + sin(x) + C.$$

Hence,

$$\int_0^{\frac{\pi}{4}} xsin(x)dx = (-x \cdot cos(x) + sin(x) + C)|_0^{\frac{\pi}{4}} = \frac{\sqrt{2}}{2}(1 - \frac{\pi}{4}).$$

(b) Compute the value of

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} cos(x)sin^2(x)dx.$$

Solution: Taking a subsitution $u = sin(x)$, then $du = cos(x)dx$, plug into the integral, we get

$$\int cos(x)sin^2(x)dx = \int u^2 du = \frac{u^3}{3} + C = \frac{sin^3(x)}{3} + C.$$

Hence,

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} cos(x)sin^2(x)dx = (\frac{sin^3(x)}{3} + C)|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{3} - \frac{\sqrt{2}}{12}.$$