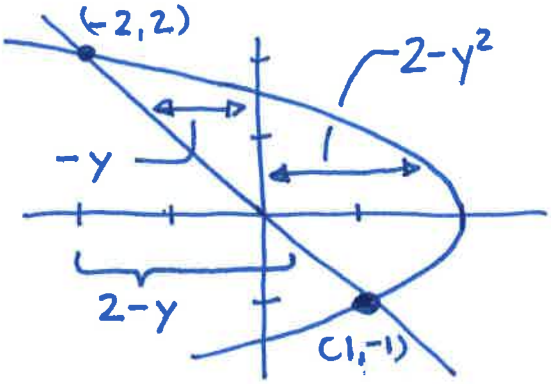
NAME: Section #:
I.D.: Instructor:

Problem 1.

(a) Find the area of the region bounded by $x+y^2=2$ and $x+y=0$.



$$\text{Area} = \int_{-1}^{2} (2-y^2)-(-y)\, dy$$

$$= \int_{-1}^{2} -y^2+y+2\, dy$$

$$= \left[-\tfrac{1}{3}y^3+\tfrac{1}{2}y^2+2y\right]_{-1}^{2}$$

$$= -\tfrac{8}{3}+2+2-\left(\tfrac{1}{3}+\tfrac{1}{2}-2\right) = 9/2$$

(b) Find the volume of the resulting solid when the same region is rotated around the line $x=-2$.

By the washer method:

$$V = \int_{-1}^{2} \pi\left((2-y^2+2)^2-(2-y)^2\right) dy$$

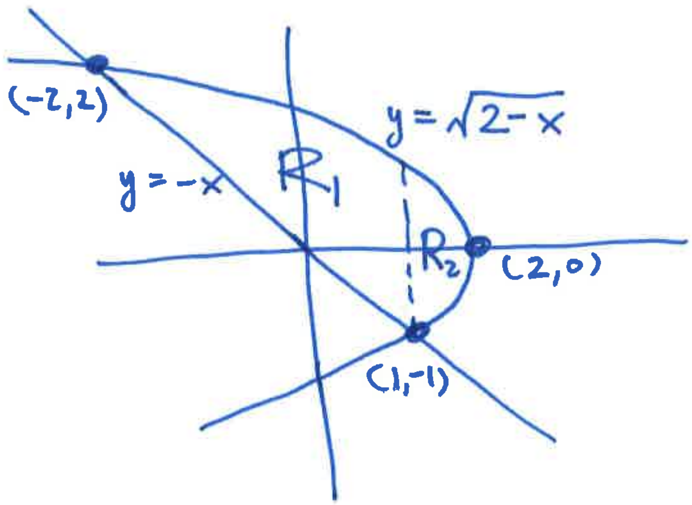
$$= \int_{-1}^{2} \pi(12+4y-9y^2+y^4)\, dy$$

$$= \pi\left[12y+2y^2-3y^3+\tfrac{1}{5}y^5\right]_{-1}^{2}$$

$$= \pi\left(\tfrac{72}{5}-\left(-\tfrac{36}{5}\right)\right)$$

$$= \frac{108\pi}{5}$$

If one is more comfortable integrating w/ respect to x, then one might do Problem 1(a) as follows:



$$\text{Area} = \text{Area}(R_1) + \text{Area}(R_2)$$

$$= \int_{-2}^{1} \sqrt{2-x} - (-x)\, dx$$

$$+ \int_{1}^{2} \sqrt{2-x} - \left(-\sqrt{2-x}\right) dx$$

$$= \int_{-2}^{1} \sqrt{2-x}\, dx + \int_{-2}^{1} x\, dx + 2\int_{1}^{2} \sqrt{2-x}\, dx$$

Make the substitution $u = 2-x$ to do the first & third integrals:

$$= -\int_{4}^{1} u^{1/2}\, du + \left[\frac{x^2}{2}\right]_{-2}^{1} - 2\int_{1}^{0} u^{1/2}\, du$$

$$= \left[\frac{2}{3}u^{3/2}\right]_{1}^{4} + \frac{1}{2} - 2 + 2\left[\frac{2}{3}u^{3/2}\right]_{0}^{1}$$

$$= \frac{2}{3}\cdot 8 - \frac{2}{3} + \frac{1}{2} - 2 + \frac{4}{3} = \frac{9}{2}$$

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Problem 3.

(a) Compute
$$\frac{\mathrm{d}}{\mathrm{d}x}\int_3^x e^{t^2}\,\mathrm{d}t.$$

$$= \frac{d}{dx}\left(\int_0^x e^{t^2}\,dt - \int_0^3 e^{t^2}\,dt \right)$$

$$= \frac{d}{dx}\left(\int_0^x e^{t^2}\,dt \right)$$ because the second term is a constant

$= e^{x^2}$ by the Fund. Thm. of Calc.

(b) Compute
$$\frac{\mathrm{d}}{\mathrm{d}x}\int_{x^2}^{x^3} e^{t^2}\,\mathrm{d}t$$

Let $f(x) = x^2$, $g(x) = x^3$,

$H(x) = \int_0^x e^{t^2}\,dt$, so

$$\int_{x^2}^{x^3} e^{t^2}\,dt = \int_0^{x^3} e^{t^2}\,dt - \int_0^{x^2} e^{t^2}\,dt$$

$$= H(g(x)) - H(f(x)).$$

We have $H'(x) = e^{x^2}$ by the Fund. Thm. of Calc, so

$$\frac{d}{dx}\int_{x^2}^{x^3} e^{t^2}\,dt = H'(g(x))\,g'(x) - H'(f(x))\,f'(x)$$

$$= e^{x^6}\cdot 3x^2 - e^{x^4}\cdot 2x$$

by the Chain Rule.