MATH 251 PROBLEMS

(1) Consider a partially ordered set X and let $Cat(X)$ be the associated categories (a unique arrow $x \to y$ for each pair $x \leq y$). Show that the product of x and y in $Cat(X)$, if exists, is the greatest lower bound of x, y. Identify similarly the coproduct.
(2) Use the previous exercise to cook up a category where products and coproducts don't always exist.
(3) Let Y be a set and $P(Y)$ be the set of all subsets of Y, partially ordered by inclusion. Identify explicitly products and coproducts in $Cat(P(Y))$.
(4) Let $A \to B$ be an abelian group homomorphism. What is the fibered product $A \times_B 0$ in elementary terms? What is the cofibered coproduct $0 \sqcap^A B$?
(5) If you have not done so, prove that a group object in *Groups* is an *abelian* group.
(6) Lang p 115 ex 1,3,4
(7) If $S \subset R$ contains no zero divisors, show that $R \to S^{-1}R$ is injective.
(8) Prove that if $p \neq q$ are distinct primes, then $\mathbb{Z}_{(p)} \not\simeq \mathbb{Z}_{(q)}$.
(9) Let M be a finitely generated R module and $S \subset R$ multiplicative. Show that $S^{-1}M = 0$ if and only if there is $d \in S$ with $dM = 0$.
(10) Lang p. 253 ex 2,3,5,6,11
(11) Determine the minimal polynomial of $\sqrt{2} + \sqrt{3}$