# MODULI OF ALGEBRAIC AND TROPICAL CURVES

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ABSTRACT. *Moduli spaces* are a geometer's obsession. A celebrated example in algebraic geometry is the space $\bar{M}_{g,n}$ of stable $n$-pointed algebraic curves of genus $g$, due to Deligne–Mumford and Knudsen. It has a delightful combinatorial structure based on *weighted graphs*.

Recent papers of Branetti, Melo, Viviani and of Caporaso defined an entirely different moduli space of *tropical curves*, which are weighted metrized graphs. It also has a delightful combinatorial structure based on weighted graphs.

One is led to ask whether there is a geometric connection between these moduli spaces. In joint work [1] with Caporaso and Payne, we exhibit a connection, which passes through a third type of geometry - nonarchimedean analytic geometry.

## REFERENCES

[1] D. Abramovich, L. Caporaso, and S. Payne, *The tropicalization of moduli space*, ArXiv 1212.0373

[2] D. Abramovich, *Moduli of algebraic and tropical curves*, ArXiv 1301.0474

*Date*: 2013.2.27.

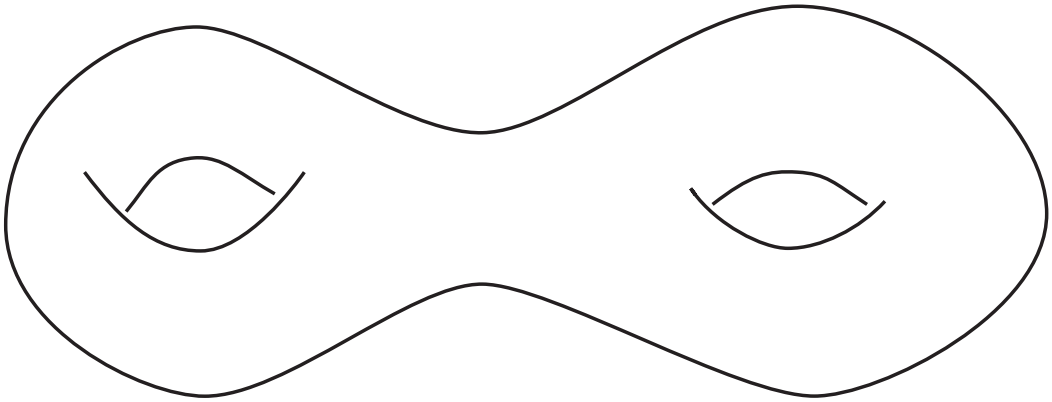


FIGURE 1. A Riemann surface of genus 2

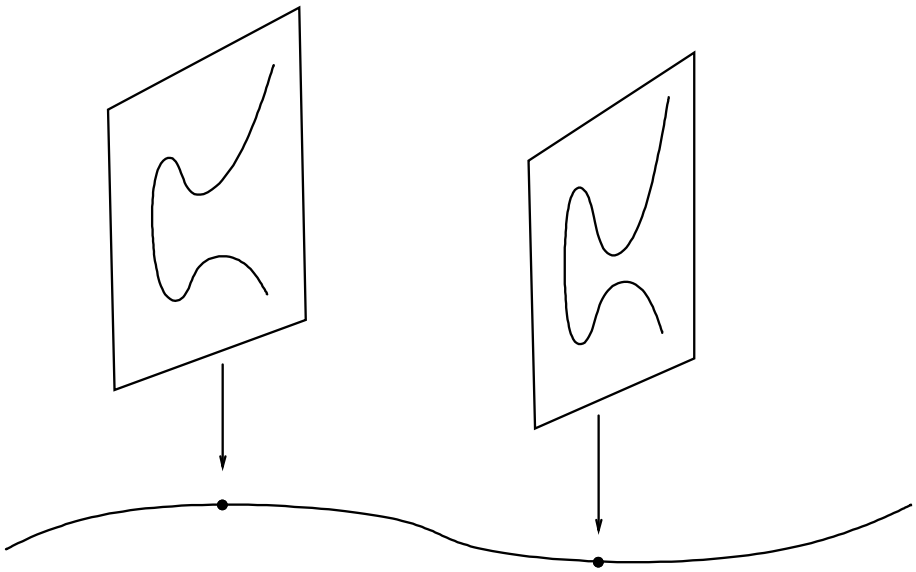


FIGURE 2. The family of elliptic curves over $\mathbb{C}$

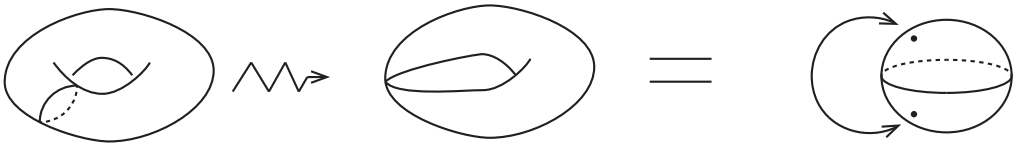


FIGURE 3. A degenerate elliptic curve as a sphere with glued points

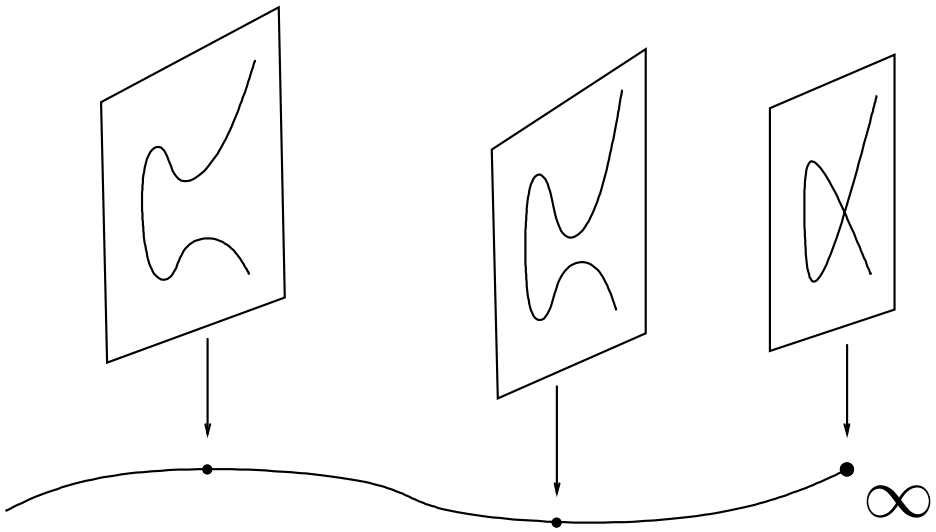



FIGURE 4. The family of elliptic curves over $\mathbb{P}^1_{\mathbb{C}}$

FIGURE 5. A degenerate Riemann surface of genus 2

FIGURE 6. A degenerate Riemann surface of genus 2

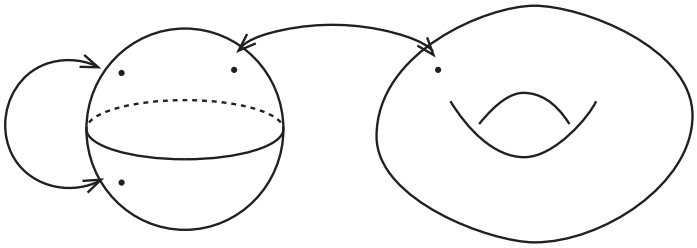


FIGURE 7. Gluing the same degenerate Riemann surface of genus 2

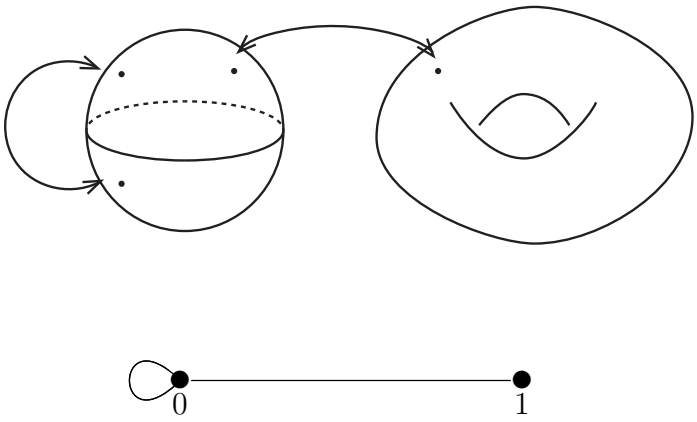



FIGURE 8. The glued curve ... and its graph

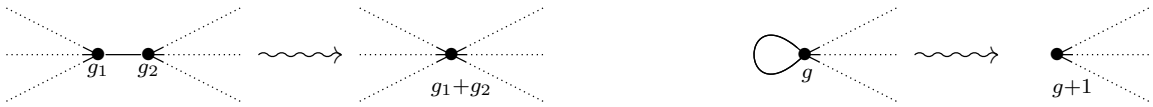



FIGURE 9. Contracting an edge ... and a loop

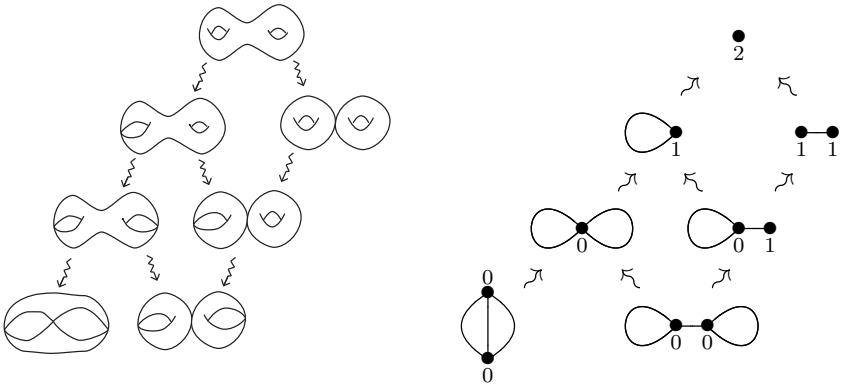



FIGURE 10. Curves in $\overline{\mathcal{M}}_2$ ... and their graphs

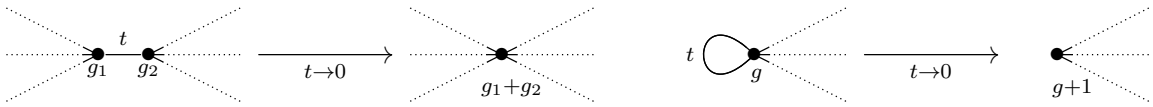



FIGURE 11. Pulling an edge ... and a loop

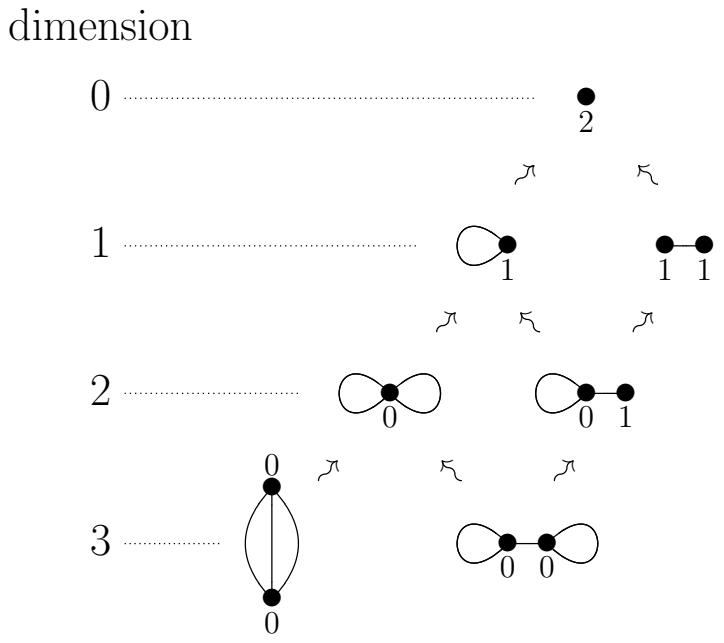
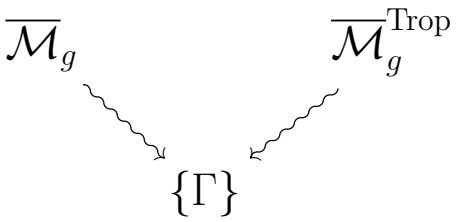
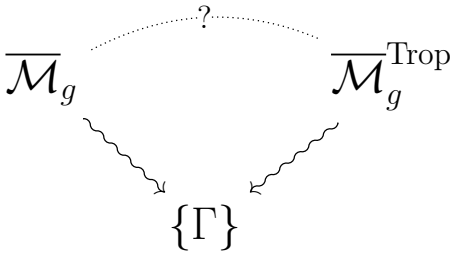



FIGURE 12. Graph contractions in genus 2

$$\mathcal{M}_{\Gamma'} \subset \overline{\mathcal{M}}_{\Gamma} \quad \Longleftrightarrow \quad \exists \text{ contraction } \ \Gamma' \to \Gamma.$$

$$\overline{\mathcal{M}^{\mathrm{Trop}}_{\Gamma'}} \supset \mathcal{M}^{\mathrm{Trop}}_{\Gamma} \quad \Longleftrightarrow \quad \exists \text{ contraction } \ \Gamma' \to \Gamma.$$


$\overline{\mathcal{M}}_g$
$\overline{\mathcal{M}}_g^{\mathrm{Trop}}$
$\{\Gamma\}$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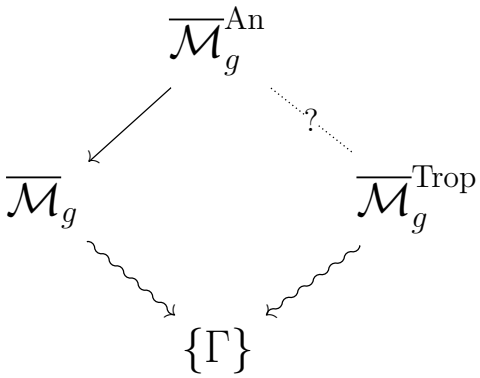
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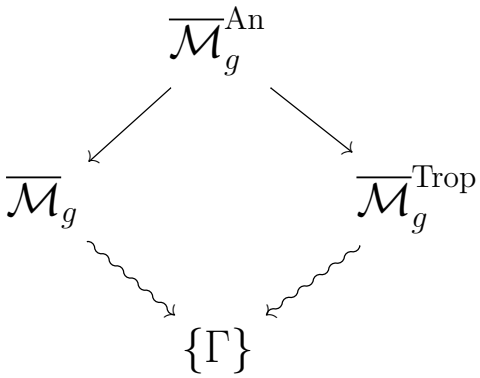
$\overline{\mathcal{M}}_g^{\mathrm{An}}$
$\overline{\mathcal{M}}_g$
$\overline{\mathcal{M}}_g^{\mathrm{Trop}}$
$\{\Gamma\}$

## References


[1] D. Abramovich, L. Caporaso, and S. Payne, *The tropicalization of moduli space*, ArXiv 1212.0373

[2] D. Abramovich, *Moduli of algebraic and tropical curves*, ArXiv 1301.0474