It was noted by Fabio Tonini that the proof using lie algebras in [1, Lemma 2.17 and Proposition 3.6] does not hold for local group schemes. While the alternative proofs we provided there using cotangent bundles hold, no alternative was provided for [2, Lemma 3.8], and in fact Tonini provided a counterexample to the argument used there. This necessitates discarding [2, Lemma 3.8] and providing an alternative proof for the only statement relying on it, [2, Proposition 3.6], which we provide below.

We proceed as in the proof of [2, Proposition 3.6] on page 415, in particular we may assume the coarse moduli spaces X and Y coincide, P and V are strictly henselian, and we have a homomorphism $\rho : \Gamma \to G$ of well-split linearly reductive group-schemes. We replace the argument on page 416 as follows:

STEP 1: it suffices to consider the case where $\rho : \Gamma \to G$ is trivial.

Assume the case where ρ is trivial holds true, and consider an arbitrary ρ. Write $K = \ker \rho$ and $Q = \Gamma/K$. Let $\mathcal{X} = [V/\Gamma]$ and $\mathcal{U} = [V/K]$, with the natural morphism $\mathcal{U} \to \mathcal{X}$. Write $U'' = U \times_{[U/Q]} U$ and $\mathcal{U}'' = \mathcal{U} \times_{\mathcal{X}} \mathcal{U}$. Since $U \to [U/Q]$ is a Q-torsor, the following diagram is cartesian

$$\begin{array}{ccccc} \mathcal{U}'' & \rightrightarrows & \mathcal{U} & \longrightarrow & \mathcal{X} \\ \downarrow & & \downarrow & & \downarrow \\ U'' & \rightrightarrows & U & \longrightarrow & [U/Q] \end{array}$$

Since $K \to G$ is trivial, the assumption implies that the composite arrow $\mathcal{U} \to \mathcal{X} \to \mathcal{Y}$ factors uniquely as $\mathcal{U} \to U \to \mathcal{Y}$. Similarly the arrow $\mathcal{U}'' \to \mathcal{X} \to \mathcal{Y}$ factors uniquely as $\mathcal{U}'' \to U'' \to \mathcal{Y}$. Commutativity implies that $\mathcal{X} \to \mathcal{Y}$ factors uniquely as $\mathcal{X} \to [U/Q] \to \mathcal{Y}$ as required.

Se we assume below that $\rho : \Gamma \to G$ is trivial, and need to prove that $\mathcal{X} \to \mathcal{Y}$ factors uniquely as $\mathcal{X} \to X \to \mathcal{Y}$.

STEP 2: it suffices to assume G is simple.

We argue by induction on the length of the Jordan-Hölder filtration of G, and assume we have a nontrivial extension $1 \to \Delta \to G \to H \to 1$. Write $\mathcal{Y} = [P/G]$ and $\mathcal{Y}_1 = [(P/\Delta)/H]$ with the natural morphism $a : \mathcal{Y} \to \mathcal{Y}_1$. Let $\psi : \operatorname{Spec} \Omega \to \mathcal{Y}$ be a geometric point, and denote by $\tilde{a} : G_\psi \to G_{a\circ\psi}$ the homomorphism of stabilizers. Then by definition $G_{a\circ\psi} \subset H$ and $\operatorname{Ker}(\tilde{a}) \subset \Delta$. **Not as lucid as I would wish to be.**

By induction the composite arrow $\mathcal{X} \to \mathcal{Y} \stackrel{a}{\to} \mathcal{Y}_1$ factors uniquely as $\mathcal{X} \to X \to \mathcal{Y}_1$.

Write $\widetilde{\mathcal{Y}} = X \times_{\mathcal{Y}_1} \mathcal{Y}$. It follows that $\widetilde{\mathcal{Y}}$ has stabiliers in Δ. By induction $\mathcal{X} \to \widetilde{\mathcal{Y}}$ factors uniquely as $\mathcal{X} \to X \to \widetilde{\mathcal{Y}}$, as needed.

Step 3: the result works for G étale.

The argument of [2, Lemma 3.8] does work for étale group schemes since the deformation and obstruction theory is trivial. In the situation to which we reduced here, everything is strictly henselian, so there is an equivalence of categories between G-torsors on $\mathcal{X}$ and G-torsors on its reduction $\mathcal{B}\Gamma_0$. But the G-torsor $P \times_{\mathcal{Y}} \mathcal{B}\Gamma_0 \to \mathcal{B}\Gamma_0$ is trivial since ρ is trivial. So $P \times_{\mathcal{Y}} \mathcal{X} \to \mathcal{X}$ is trivial. Taking a section we get a factorization $\mathcal{X} \to P \to \mathcal{Y}$ of $\mathcal{X} \to \mathcal{Y}$, and since P is representable it factors as $\mathcal{X} \to X \to P \to \mathcal{Y}$. It remains to note that $\mathcal{X} \to X \to \mathcal{Y}$ is independent of the choice of section and is unique. **This needs a word**

Step 4: the result works for $\mathcal{Y} = X \times \mathcal{B}\mu_p$.

Here the X-morphism $\mathcal{X} \to \mathcal{Y}$ is given by a μ_p-torsor $R_{\mathcal{X}} \to \mathcal{X}$, equivalently a line bundle L with a trivialization of $L^{\otimes p}$. Since $\Gamma \to \mu_p$ is trivial, the action of the stabilizer of any geometric point on the fiber of L is trivial. By [3, Proposition 6.2] the pullback functor gives an equivalence of categories between line bundles on X and line bundles on $\mathcal{X}$ such that the stabilizers act trivially on their fibers. It follows that the μ_p-torsor $R_{\mathcal{X}} \to \mathcal{X}$ is the pullback of a μ_p-torsor $R_X \to X$, giving the desired factorization $\mathcal{X} \to X \to \mathcal{Y}$.

Step 5: the result works for $G = \mu_p$.

Unlike the claim in [2, Lemma 3.8], we will change the μ_p-torsor $P \to \mathcal{Y}$. We claim that there is a μ_p torsor $P' \to \mathcal{Y}$ such that P' is representable, and its pullbacks to $\mathcal{X}$ is trivial. As in the previous step, a section $\mathcal{X} \to P' \times_{\mathcal{Y}} \mathcal{X}$ gives a morphism $\mathcal{X} \to P'$ which factors through X, giving the required factorization $\mathcal{X} \to X \to \mathcal{Y}$. **Still need uniqueness.** So it remains to prove this claim.

By definition torsor $P \to \mathcal{Y}$ is the pullback of a nontrivial μ_p-torsor χ on $\mathcal{B}\mu_p$. By the previous step, the pullback of χ along $\mathcal{X} \to \mathcal{Y} \to \mathcal{B}\mu_p$, namely $P \times_{\mathcal{Y}} \mathcal{X}$, is also the pullback of a torsor $R \to X$ along $\mathcal{X} \to X$. Denote $P' = P \otimes R_{\mathcal{Y}}^{-1}$. Then P' is still representable since the action of μ_p on the stabilizers is nontrivial, and by definition its pullback to $\mathcal{X}$ is trivial, and the claim follows.

References

AOV1 [1] Abramovich, Dan; Olsson, Martin; Vistoli, Angelo. *Tame stacks in positive characteristic.* Ann. Inst. Fourier (Grenoble) 58 (2008), no. 4, 1057–1091.

AOV2 [2] Abramovich, Dan; Olsson, Martin; Vistoli, Angelo. *Twisted stable maps to tame Artin stacks.* J. Algebraic Geom. 20 (2011), no. 3, 399–477.

Olsson [3] Olsson, Martin. *Integral models for moduli spaces of G-torsors.* Ann. Inst. Fourier (Grenoble) 62 (2012), no. 4, 1483–1549.