It was noted by Fabio Tonini that the proof using lie algebras in [1, Lemma 2.17 and Proposition 3.6] does not hold for local group schemes. While the alternative proofs we provided there using cotangent bundles hold, no alternative was provided for [2, Lemma 3.8], and in fact Tonini provided a counterexample to the argument used there. This necessitates discarding [2, Lemma 3.8] and providing an alternative proof for the only statement relying on it, [2, Proposition 3.6], which we provide below.

We proceed as in the proof of [2, Proposition 3.6] on page 415, in particular we may assume the coarse moduli spaces X and Y coincide, P and V are strictly henselian, and we have a homomorphism $\rho : \Gamma \to G$ of well-split linearly reductive group-schemes. We replace the argument on page 416 as follows:

STEP 1: it suffices to consider the case where $\rho : \Gamma \to G$ is trivial.

Assume the case where ρ is trivial holds true, and consider an arbitrary ρ. Write $K = \ker \rho$ and $Q = \Gamma/K$. Let $\mathcal{X} = [V/\Gamma]$ and $\mathcal{U} = [V/K]$, with the natural morphism $\mathcal{U} \to \mathcal{X}$. Write $U'' = U \times_{[U/Q]} U$ and $\mathcal{U}'' = \mathcal{U} \times_{\mathcal{X}} \mathcal{U}$. Since $U \to [U/Q]$ is a Q-torsor, the following diagram is cartesian

$$\begin{array}{ccccc} \mathcal{U}'' & \rightrightarrows & \mathcal{U} & \longrightarrow & \mathcal{X} \\ \downarrow & & \downarrow & & \downarrow \\ U'' & \rightrightarrows & U & \longrightarrow & [U/Q] \end{array}$$

Since $K \to G$ is trivial, the assumption implies that the composite arrow $\mathcal{U} \to \mathcal{X} \to \mathcal{Y}$ factors uniquely as $\mathcal{U} \to U \to \mathcal{Y}$. Similarly the arrow $\mathcal{U}'' \to \mathcal{X} \to \mathcal{Y}$ factors uniquely as $\mathcal{U}'' \to U'' \to \mathcal{Y}$. Commutativity implies that $\mathcal{X} \to \mathcal{Y}$ factors uniquely as $\mathcal{X} \to [U/Q] \to \mathcal{Y}$ as required.

Se we assume below that $\rho : \Gamma \to G$ is trivial, and need to prove that $f : \mathcal{X} \to \mathcal{Y}$ factors uniquely as $\mathcal{X} \to X \to \mathcal{Y}$, where $q : \mathcal{X} \to X := V/\Gamma$ is the coarse moduli space of $\mathcal{X}$.

STEP 2: proof when ρ is trivial.

By [3, 6.4] pullback defines an equivalence of categories between G-torsors over X and G-torsors P on $\mathcal{X}$ such that for every geometric point $\bar{x} \to \mathcal{X}$ the induced action of the stabilizer $\Gamma_{\bar{x}}$ on $P_{\bar{x}}$ is trivial. Now giving a factorization $g : X \to \mathcal{Y}$ of f is equivalent to giving a pair $(T \to X, \bar{\sigma} : T \to U)$, where T/X is a G-torsor and $\sigma : T \to U$ is a G-equivariant map. By the preceding observation, such a pair is in turn equivalent to a pair $(P \to \mathcal{X}, \sigma : P \to U)$, where $P/\mathcal{X}$ is a G-torsor such that the stabilizer action is trivial at every point and σ is a G-equivariant map (note that if T/X is the corresponding torsor then since T is flat over X the scheme T is the coarse moduli space of $P = T \times_X \mathcal{X}$ and the map σ factors uniquely through a necessarily G-equivariant map $\bar{\sigma} : T \to U$). The existence and uniqueness of g therefore follows from noting that the stabilizer action on the pullback along f of the G-torsor $U \to \mathcal{Y}$ is trivial since the map ρ is trivial.

References

AOV1 [1] Abramovich, Dan; Olsson, Martin; Vistoli, Angelo. *Tame stacks in positive characteristic.* Ann. Inst. Fourier (Grenoble) 58 (2008), no. 4, 1057–1091.

AOV2 [2] Abramovich, Dan; Olsson, Martin; Vistoli, Angelo. *Twisted stable maps to tame Artin stacks.* J. Algebraic Geom. 20 (2011), no. 3, 399–477.

Olsson [3] Olsson, Martin. *Integral models for moduli spaces of G-torsors.* Ann. Inst. Fourier (Grenoble) 62 (2012), no. 4, 1483–1549.