Resolution by weighted blowing up

Dan Abramovich, Brown University
Joint work with Michael Tëmkin and Jarosław Włodarczyk

Also parallel work by M. McQuillan with G. Marzo

Logarithmic Enumerative Geometry and Mirror Symmetry

MFO, June 18, 2019

Main result

Theorem (ℵ-T-W, MM, "weighted Hironaka", characteristic 0)

There is a functor F associating to a singular subvariety $X \subset Y$ embedded with pure codimension c in a smooth variety Y, a center $\bar{J}$ with blowing up $Y' \to Y$ and proper transform $(X' \subset Y') = F(X \subset Y)$ such that $\operatorname{maxinv}(X') < \operatorname{maxinv}(X)$. *In particular, for some n the iterate $(X_n \subset Y_n) := F^{\circ n}(X \subset Y)$ of F has X_n smooth.*

Main result

Theorem ($\aleph$-T-W, MM, "weighted Hironaka", characteristic 0)

There is a functor F associating to a singular subvariety $X \subset Y$ embedded with pure codimension c in a smooth variety Y, a center $\bar{J}$ with blowing up $Y' \to Y$ and proper transform $(X' \subset Y') = F(X \subset Y)$ such that $\operatorname{maxinv}(X') < \operatorname{maxinv}(X)$. In particular, for some n the iterate $(X_n \subset Y_n) := F^{\circ n}(X \subset Y)$ of F has X_n smooth.

- This is functorial for smooth surjective morphisms.

Main result

Theorem (ℵ-T-W, MM, "weighted Hironaka", characteristic 0)

There is a functor F associating to a singular subvariety $X \subset Y$ embedded with pure codimension c in a smooth variety Y, a center $\bar{J}$ with blowing up $Y' \to Y$ and proper transform $(X' \subset Y') = F(X \subset Y)$ such that $\mathrm{maxinv}(X') < \mathrm{maxinv}(X)$*. In particular, for some n the iterate $(X_n \subset Y_n) := F^{\circ n}(X \subset Y)$ of F has X_n smooth.*

- This is functorial for smooth surjective morphisms.
- Example: blowing up $x^2 - y^2z$ at the origin, gives in the z-chart $x = x_3z,\ y = y_3z$ the equation $z^2(x_3^2 - y_3^2z) = 0$.

Main result

Theorem (ℵ-T-W, MM, "weighted Hironaka", characteristic 0)

There is a functor F associating to a singular subvariety $X \subset Y$ embedded with pure codimension c in a smooth variety Y, a center $\bar{J}$ with blowing up $Y' \to Y$ and proper transform $(X' \subset Y') = F(X \subset Y)$ such that $\operatorname{maxinv}(X') < \operatorname{maxinv}(X)$. *In particular, for some n the iterate $(X_n \subset Y_n) := F^{\circ n}(X \subset Y)$ of F has X_n smooth.*

- This is functorial for smooth surjective morphisms.
- Example: blowing up $x^2 - y^2 z$ at the origin, gives in the z-chart $x = x_3 z, \; y = y_3 z$ the equation $z^2(x_3^2 - y_3^2 z) = 0$.
- However, blowing up $\bar{J} = (x^{1/3}, y^{1/2}, z^{1/2})$ gives in the z-chart $x = w^3 x_3, y = w^2 y_3, z = w^2$ the equation $w^6(x_3^2 - y_3^2)$,

Main result

Theorem (ℵ-T-W, MM, "weighted Hironaka", characteristic 0)

There is a functor F associating to a singular subvariety $X \subset Y$ embedded with pure codimension c in a smooth variety Y, a center $\bar{J}$ with blowing up $Y' \to Y$ and proper transform $(X' \subset Y') = F(X \subset Y)$ such that $\operatorname{maxinv}(X') < \operatorname{maxinv}(X)$. In particular, for some n the iterate $(X_n \subset Y_n) := F^{\circ n}(X \subset Y)$ of F has X_n smooth.

- This is functorial for smooth surjective morphisms.
- Example: blowing up $x^2 - y^2 z$ at the origin, gives in the z-chart $x = x_3 z,\ \ y = y_3 z$ the equation $z^2(x_3^2 - y_3^2 z) = 0$.
- However, blowing up $\bar{J} = (x^{1/3}, y^{1/2}, z^{1/2})$ gives in the z-chart $x = w^3 x_3, y = w^2 y_3, z = w^2$ the equation $w^6(x_3^2 - y_3^2)$,
- and the weights dropped: $(2, 2) < (2, 3, 3)$.

The end

Thank you for your attention