Description of admissibility

**Definition**

J is \( \mathcal{I} \)-admissible if \( J \leq \nu(\mathcal{I}) \).

**Lemma**

This is equivalent to \( \mathcal{I} \mathcal{O}_{Y'} = E^\ell \mathcal{I}' \), with \( J = \bar{J}^\ell \) and \( \mathcal{I}' \) an ideal.

- Indeed, on \( Y' \) the center \( J \) becomes \( E^\ell \), in particular principal.
Description of admissibility

Definition

\( J \) is \( \mathcal{I} \)-admissible if \( J \leq v(\mathcal{I}) \).

Lemma

This is equivalent to \( \mathcal{I} \mathcal{O}_{Y'} = E^\ell \mathcal{I}' \), with \( J = \bar{J}^\ell \) and \( \mathcal{I}' \) an ideal.

- Indeed, on \( Y' \) the center \( J \) becomes \( E^\ell \), in particular principal.
- So on \( Y' \), we have \( J \leq v(\mathcal{I}) \iff E^\ell \supseteq \mathcal{I} \mathcal{O}_{Y'} \).
- This is more subtle in Quek’s theorem!
**Description of admissibility**

**Definition**

$J$ is $\mathcal{I}$-admissible if $J \leq v(\mathcal{I})$.

**Lemma**

This is equivalent to $\mathcal{I}O_{Y'} = E^\ell \mathcal{I}'$, with $J = \overline{J}^\ell$ and $\mathcal{I}'$ an ideal.

- Indeed, on $Y'$ the center $J$ becomes $E^\ell$, in particular principal.
- So on $Y'$, we have $J \leq v(\mathcal{I}) \iff E^\ell \supseteq \mathcal{I}O_{Y'}$. ♠
- This is more subtle in Quek’s theorem!
- Write $J = (x_1^{a_1}, \ldots, x_k^{a_k})$ and $\mathcal{I} = (f_1, \ldots, f_m)$.
- Expand $f_i = \sum c_\alpha x_1^{\alpha_1} \cdots x_n^{\alpha_n}$.
- $J < v(\mathcal{I}) \iff v_J(f_i) \geq 1$ for all $i$
Description of admissibility

**Definition**

\( J \) is \( \mathcal{I} \)-admissible if \( J \leq v(\mathcal{I}) \).

**Lemma**

This is equivalent to \( \mathcal{I} \mathcal{O}_{Y'} = E^l \mathcal{I}', \) with \( J = \bar{J}^l \) and \( \mathcal{I}' \) an ideal.

- Indeed, on \( Y' \) the center \( J \) becomes \( E^l \), in particular principal.
- So on \( Y' \), we have \( J \leq v(\mathcal{I}) \) \( \iff \) \( E^l \supseteq \mathcal{I} \mathcal{O}_{Y'} \).
- This is more subtle in Quek’s theorem!
- Write \( J = (x_1^{a_1}, \ldots, x_k^{a_k}) \) and \( \mathcal{I} = (f_1, \ldots, f_m) \).
- Expand \( f_i = \sum c_\alpha x_1^{\alpha_1} \cdots x_n^{\alpha_1} \).
- \( J < v(\mathcal{I}) \) \( \iff \) \( v_J(f_i) \geq 1 \) for all \( i \)
- \( \iff \) \( \sum \frac{\alpha_j}{a_j} \geq 1 \) for all \( i \) and \( \alpha \) such that \( c_\alpha \neq 0 \).
Consequences

- $J$ is $\mathcal{I}_1, \mathcal{I}_2$-admissible $\Rightarrow$ $J$ is $\mathcal{I}_1 + \mathcal{I}_2$-admissible.
- $J$ is $\mathcal{I}$-admissible $\Rightarrow$ $J^a$ is $\mathcal{I}^a$-admissible.
- $J$ is $\mathcal{I}$-admissible $\Rightarrow$ $J^{1-\frac{1}{a_1}}$ is $\mathcal{D}(\mathcal{I})$-admissible.
Consequences

- $J$ is $\mathcal{I}_1, \mathcal{I}_2$-admissible $\Rightarrow$ $J$ is $\mathcal{I}_1 + \mathcal{I}_2$-admissible.
- $J$ is $\mathcal{I}$-admissible $\Rightarrow$ $J^a$ is $\mathcal{I}^a$-admissible.
- $J$ is $\mathcal{I}$-admissible $\Rightarrow$ $J^{1-\frac{1}{a_1}}$ is $\mathcal{D}(\mathcal{I})$-admissible.

Proof.

$$\nu_J \left( \frac{\partial x^\alpha}{\partial x_j} \right) = \sum \frac{\alpha_i}{a_i} - \frac{1}{a_j} \geq 1 - \frac{1}{a_1}.$$
Consequences

- $J$ is $\mathcal{I}_1, \mathcal{I}_2$-admissible $\Rightarrow$ $J$ is $\mathcal{I}_1 + \mathcal{I}_2$-admissible.
- $J$ is $\mathcal{I}$-admissible $\Rightarrow$ $J^a$ is $\mathcal{I}^a$-admissible.
- $J$ is $\mathcal{I}$-admissible $\Rightarrow$ $J^{1 - \frac{1}{a_1}}$ is $\mathcal{D}(\mathcal{I})$-admissible.

Proof.

$$v_J \left( \frac{\partial x^\alpha}{\partial x_j} \right) = \sum \frac{\alpha_i}{a_i} - \frac{1}{a_j} \geq 1 - \frac{1}{a_1}.$$  ♠

Combining:

Proposition

A center $J$ is $\mathcal{I}$-admissible if and only if $J^{(a_1 - 1)!}$ is $\mathcal{C}(\mathcal{I}, a_1)$-admissible.