Properties of the invariant

**Theorem (MC-invariance [Włodarczyk, Kollár])**

Given maximal contacts $x_1, x_1'$ and a common extension to regular systems of parameters $(x_1, x_2, \ldots, x_n)$ and $(x_1', x_2, \ldots, x_n)$, there are étale $\pi, \pi' : \tilde{Y} \rightarrow Y$ such that

$$
\pi^* x_1 = \pi'^* x_1'
$$

$$
\pi^* x_2 = \pi'^* x_2
$$

$$
\vdots
$$

$$
\pi^* C(I, a_1) = \pi'^* C(I, a_1).
$$

**Proposition**

$inv_p$ is functorial, well-defined, upper-semi-continuous.
$\text{inv}_p$ is functorial, well-defined, upper-semi-continuous.

- $a_1 = \text{ord}$, $\mathcal{D}(\mathcal{I})$ are functorial for smooth maps $\Rightarrow C(\mathcal{I}, a_1)$ is functorial,
$\text{inv}_p$ is functorial, well-defined, upper-semi-continuous.

- $a_1 = \text{ord}$, $\mathcal{D}(\mathcal{I})$ are functorial for smooth maps $\Rightarrow C(\mathcal{I}, a_1)$ is functorial,
- so by induction $\text{inv}_p(\mathcal{I}, x_1, \ldots, x_n)$ is functorial.

Note that $\text{inv}_p(\mathcal{I}, x_1, \ldots, x_n) = \text{inv}_p(\mathcal{I}, x_1, x_2 + tx_1, \ldots, x_n + tx_1)$.

Choosing appropriate $t$ we have $(x_1', x_2 + tx_1', \ldots, x_n + tx_1')$ system of parameters.


By induction and functoriality $a_2, \ldots, a_n$ well defined. $(a_2, \ldots, a_n)$ USC on $V(x_1)$, containing the maximal locus of $a_1$,
so $\text{inv}_p$ USC.
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- \(a_1 = \text{ord}, \ D(\mathcal{I})\) are functorial for smooth maps \(\Rightarrow C(\mathcal{I}, a_1)\) is functorial,
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- Choosing appropriate $t$ we have $(x'_1, x_2 + tx_1, \ldots, x_n + tx_1)$ system of parameters.
$\text{inv}_p$ is functorial, well-defined, upper-semi-continuous.

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