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Introduction by **Gregory Benford**

The Reverend Edwin Abbott Abbott, M.A., D.D., headmaster of the City of London School, was a bit odd.

Respected, well liked, he led a strictly regular life, as proper as a parallelogram. He had published quite a few conventional books with titles like *Through Nature to Christ*, *Parables for Children*, and *How to Tell the Parts of Speech.* These did not prepare the world for his sudden excursion into the fantastic, in 1884. *Flatland* has proved his only hedge against oblivion, an astonishingly prescient fantasy of mathematics.

Abbott's oddity begins with his repeated name, which a mathematical with might see as A times A or A Squared, A^2. In Abbott's book, the protagonist is A Square, a much troubled spirit. Liberated into another character, Abbott seems to have broken out of his cover as a prim reverend, and poured out his feelings.

The book has a curiously obsessive quality, which perhaps accounts for its uneasy reception. Reviewers termed it "soporific," "prolix," "mortally tedious," "desperately facetious," while others found it "clever," "fascinating," "never been equaled for clarity of thought," and "mind broadening," and even likened it to *Gulliver's Travels.* This last comparison is just, because beneath the math drolleries lurks a penetrating satire of Victorian society.

A Square's society is as constrained as the prim Victorian's. Women are not full figures but mere lines. Soldiers are triangles with sharp points, adept at stabbing. The more sides, the higher the status, so hexagons outrank squares, and the high priests are perfect circles.

In a delicious irony, the upper classes are polygons with equal sides–but their views certainly do not embrace equality. Mathematicians term equal-sided figures "regular," and in nineteenth century terms, proper upper class polygons are of the regular sort.

A Square learns that his view of the world is too narrow. There is a third dimension, grander and exciting, but his hidebound fellows cannot see it. This opening-out is the central imaginative event of the novel, Abbott echoing an emergent idea.

In the late nineteenth century higher dimensions were fashionable. Mathematicians had laid the foundations for rigorous work in higher-dimensional space, and physicists were about to begin using four-dimensional spacetime. Twenty centuries after Euclid, the mathematician Bernhard Riemann took a great leap in 1854, liberating the idea of dimensions from the constraints of our spatial senses. He argued that ever since Rene Descartes had described spaces with algebra, the path to discussing higher dimensions had been clear, but unwalked.

Descartes' analytic geometry defined lines as things described by one set of coordinates, or distances along one axis. A plane needed two independent coordinate sets, a solid took three. With coordinates one could map an object, defining it quantitatively: not "Chicago is over that hill," but "Chicago is fifteen miles that way." This appealed more to our logical capacity, and less to our sensory experience.

Riemann described worlds of equal logical possibility, with dimensions ranging from one to infinity. They were not spatial in the ordinary sense. Instead, Riemann took dimension to refer to conceptual spaces, which he named manifolds.

This wasn't merely a semantic change. Weather, for exmaple, depends on several variables–say, n–like temperature, pressure, wind velocity, time of day, etc. One could represent the weather as a moving point in an n-dimensional space. A plausible model of everyday weather needs about a dozen variables, and to visualize it means seeing curves and surfaces in a 12 dimensional world. No wonder we understand the motions of planets (which even Einstein needed only four dimensions to describe), but not the weather.

Riemann revolutionized mathematics and his general ideas diffused into our culture. By 1880, C.H. Hinton had pressed the issue by building elaborate models, to further his extradimensional intuition; he tried to explain ghosts as higher-dimensional apparitions. Pursuing the analogy, he wrote of a fourth-dimensional God from who nothing could be hidden. The afterlife, then, allowed spirits to move along the time dimension, reliving and reassessing moments of life. Spirits from hyperspace were the subject of J.K.F. Zollner's 1878 *Transcendental Physics*, which envisioned them moving everywhere by short-cut loops through the fourth dimension.

Mystics responded to the fashion by imagining that God, souls, angels and any other theological beings resided as literal beings of mass ("hypermatter") in four-space. This neatly explains why they can appear anywhere they like, and God can be everywhere simultaneously, the way we can look down on a Flatland and perceive it as a whole. Some found such transports of the imagination inspiring, while others thought them crass and far too literal. I am unaware of Abbott himself ever subscribing to such beliefs.

Still, Abbott and his adventuresome Square longed for the strange. More than any other writer, Abbott coined the literary currency of dimensional metaphor. By having a point of view which is literally above it all, surveying the follies of a two-dimensional plane, Abbott can adroitly satirize the staid rigidities of his Victorian world. (Perhaps this is why he first published *Flatland* under a pseudonym.)

"Irregulars" are cruelly executed, for example. Do they stand for foreigners? Gypsies? Cripples? We are left to fill in some blanks, but the overall shape of the plot is clear–flights of fancy are punished, and A Square does not finish happily.

At a deeper level, the book harks toward deep scientific issues, and the difficulty of comprehending a physical reality beyond our immediate senses. This is the great them of modern physics. The worlds of relativity and the quantum are beyond the rough-and-ready ideas we chimpanzees have built into us, from our distant ancestors' experience of throwing stones and poking sticks on African plains.

Still deeper in this fanciful narrative, the good Reverend tries to speak indirectly of intense spiritual experience. The trip into the higher realm of three dimensions is a fine metaphor for a mystical encounter.

The thrust of the deceptively simple narrative is to make us examine our basic assumptions. After all, our visual perceptions of the world are two-dimensional patterns, yet we somehow know how to see three-dimensionality. One knows instantly the difference between a ball and flat disk by their shading in available light. Objects move in front of each other, like a woman walking by a wall. We automatically discount a possible interpretation–that the woman has somehow dissolved the wall for an instant as she passes. Instead, we see here in her three-dimensionality. The eye has learned the world's geometry and discards any other scheme.

A Square learns this lesson early as he first visits Lineland in a dream. The only distinction the natives can have is in their length. They see each other as points, since they move along the same universal straight line. They estimate how far away others are by their acute sense of hearing, picking up the difference between a bass left voice and a tenor right; the time lag in arrival tells the distance. The King is longest, men next, then boys are stubby lines. Women are mere points, of lower status. Their views of each other are partial and instinctive. They never dream of how narrowly they see their world.

This set the stage for A Square's conceptual blow-out when a Sphere visits him and yanks him up into the hallucinogenic universe of three dimensions. Its realities are surrealistic. A Square struggles to fathom what for us is instinctive.

We take for granted the reality of three dimensions, but what is the reality of two dimensions? Would flatlanders have physical presence in our world–that is, could we perceive a two-dimensional universe embedded in our own? Could we yank them up into our world?

Flatlanders could be as immaterial as shadows, mere patters in our view. If an isosceles triangle soldier cut your throat it would not hurt. Abbott did not consider this in his first edition, but in the second he says that A Square eventually believes that flatlanders have a small but real height in our universe. A Sqaure discusses this with the ruler of Flatland:

> I tried to prove to him that he was "high," as well as long and broad, although he did not know it. But what was his reply? "You say I am 'high'; measure my 'high-ness' and I will believe you." What could I do? I met his challenge!

If flatlanders were even quite thick, they would not be able to tell, if in that direction they had no ability to move or did not vary. Height *as a concept* would like beyond their knowable range. Or if they did vary in height, but could not directly see this, they might ascribe the differences to qualitative features like charisma or character or "presence." There would be rather mysterious forces at work in their world, the Platonic shadows of a higher, finer reality.

If a flatlander soldier of genuine physical thickness attacked, it would cut us like a knife. Otherwise, it could not impinge upon us. We would remain oblivious to all events in the lesser dimensions.

In a sense, a truly two-dimensional flatlander faces a similar problem if it tries to digest food. A simple alimentary canal from stem to stern of, say, a circle would bisect it. To keep itself intact, a circle would have to digest by enclosing whatever it used for food in pockets, opening one and passing food to the next like a series of locks in a canal, until eventually it excreted at the far end.

This is typical of the problems engaged by thinking in another dimension. Looking downward at lower dimensions is easy. Looking up strains us.

Visualizing the fourth dimension has preoccupied many geometers. A cube in 4D is called a tesseract. One way to think of it is to look into a cube and realize that by perspective, you see the far end as a square. Diagonals (the cube edges) lead to the outer "corners" of a larger square–the cube face you're looking through. Now go to a 4D analogy. A hypercube is one small cube, sitting in the middle of a large cube, connected to it by diagonals. Or rather, that is how it would look to us, lowly 3D folk.

Cutting a hypercube in the right way allows one to unfold it and reform it into a 3D patter of eight cubes. One choice looks like a sort of 3D cross. Salvador Dali used this as a crucifix in his 1954 painting *Christus Hypercubus.* Robert Heinlein gave this a twist with "And He Built a Crooked House," in which a house built to this pattern folds back up, during an earthquake, into a true hypercube, trapping the inhabitants in four dimensions.

Rudy Rucker, mathematician and science fiction author, has taken A Sqaure and Flatland into myriad fresh adventures. I met Rucker once in the 1980s and found him much like his fictional narrators, inventive and wild, with a cerebral spin on the world, a place he found only apparently commonplace. His *The Sex Sphere* (1983) satirizes dimensional intrusions, many short storeis develop ideas only latent in *Flatland*, and his short story "Message found in a Copy of Flatland" details how a figure much like Rucker himself returns to Abbott's old haunts and finds the actual portal into that world, in the basement of a Pakistani restaurant. He finds that the triangular soldiers can indeed cut intruders from higher dimensions, and flatlanders are tasty when he gets hungry. As a sendup of the original it is pointed and funny.

In science fiction there have been many stories about creatures from the fourth dimension invading ours, generally with horrific results. While we puzzle over whether an unseen fourth dimension exists, modern physics has used the idea in the Riemannian manner, to expand our conceptual underpinnings. Riemann saw a mathematical theme of conceptual spaces, not merely geometrical ones. Physics has taken this idea and run with it.

Abbott's solving the problem of flatlander physical reality by adding a tiny height to them was strikingly prescient. Some of the latest quantum field theories of cosmology begin with extra dimensions beyond three, and then "roll up" the extras so that they are unobservably small–perhaps a billion times more tiny than an atom. Thus we are living in a universe only apparently three-dimensional; infinitesimal but real dimensions lurk all about us. There actually are *eighteen* dimensions in all!

How did this surrealistically bizarre idea come about? From considering the form and symmetries of abstruse equations. In such chilly realms, beauty is often our only guide. The embarrassment of dimensions in some theories arises from a clarity in starting with a theory which looks appealing, then hiding the extra dimensions from actually acting in our physical world. This may seem an odd way to proceed, but it has a history.

The greatest fundamental problem of physics in our time has been to unite the two great fundamental theories of the century, general relativity and quantum mechanics, into a whole, unified view of the world. In cosmology, where gravity dominates all forces, general relativity rules. In the realm of the atom, quantum processes call the tune.

They do not blend. General relativity is a "classical" theory in that it views matter as particles, with no quantum uncertainties built in. Similarly, quantum mechanics cannot include gravity in a "natural" way.

Here "natural" means in a fashion which does not violate our sense of how equations should look, their beauty. Aesthetic considerations are very important in science, not just in physics, and they are the kernel of many theories. There have even been mathematical cosmologies which begin with a two-dimensional expanding universe, and later jump to 3D, for unexplained reasons.

Einstein wove space and time together to produce the first true theory of the entire cosmos. He had first examined a spacetime which is "flat," that is, untroubled by curves and twists in the axes which determine coordinates. This was his 1905 special theory of relativity. He drew upon ideas which Abbott had already used. The eminent British journal *Nature* published in 1920 a comparison of Abbott's prophetic theme:

(Dr. Abbott) asks the reader, who has consciousness of the third dimension, to imagine a sphere descending upon the plane of Flatland and passing through it. How will the inhabitants regard this phenomenon? ... Their experience will be that of a circular obstacle gradually expanding or growing, and then contracting, and they will attribute to *growth in time* what the external observer in three

> dimensions assigns to motion in the third dimension. Transfer this analogy to a movement of the fourth dimension through three-dimensional space. Assume the past and future of the universe to be all depicted in four-dimensional space and visible to any being who has consciousness of the fourth dimension. If there is motion of our three-dimensional space relative to the fourth dimension, all the changes we experience and assign to the flow of time will be due simply to this movement, the whole of the future as well as the part always existing in the fourth dimension.

In special relativity, distance in spacetime is not the simple result we know from rectangular geometry. In the ordinary Euclidean geometry everyone learns in school, if ”d” means a small change and the coordinates of space are called x, y and z, then we find a small length (ds) in our space by adding the squares of each length, so that

$$(\mathrm{ds})^2 = (\mathrm{dx})^2 + (\mathrm{dy})^2 + (\mathrm{dz})^2$$

Contrast special relativity, in which a small distance in spacetime adds a length given by dt, a small change in time, multiplied by the speed of light, c:

$$(\mathrm{ds})^2 = (\mathrm{dx})^2 + (\mathrm{dy})^2 + (\mathrm{dz})^2 \ (\mathrm{cdt})^2$$

The trick is that the extra length (cdt) is subtracted, not added. This simple difference leads to a whole restructuring of the basic geometry. The mathematician Minkowski shoed this some years after Einstein formulated special relativity.

A thicket of confusions lurks here. Reflect that the total small (or differential, in mathematical language) length is (ds), found by taking the square root of the above equation. But if (cdt) is greater than the positive (first three) terms, then (ds) is an imaginary number! What can this mean? Physically, it means the rules for moving in this four-dimensional (4D) space are complex and contrary to our 3D intuitions. Different kinds of curves are called ”spacelike” and ”timelike,” because they have very different physical properties.

Einstein was fond of saying that he viewed the world as 4D, with people existing in it simultaneously. This meant that in 4D the whole of life of a person (their ”world-line”) was on view. Life was eternal, in a sense–a cosmic distancing available mostly to mathematicians and lovers of abstraction.

Einstein's was the first major scientific use of time as an added dimension, though literature had gotten there first. By 1895 the widespread use of dimensional imagery led H.G. Wells to depict time as just another axis of space-like cosmos, so that one could move forward and back along it. In a sense Wells's use domesticated the fourth dimension, relieving it of genuinely jarring strangeness, and ignoring the possibility of time paradox, too.

Einstein's theory contrasts strongly with visions such as Wells' in *The Time Machine*, which treats motion along the (dt) axis as very much like taking a train to the future, then back. In Einstein's geometry, only portions of space can be reached at all without violating causality (the ”light cone” within which two points can be connected by a single beam of light). Paradoxes can abound.

Logical twists have inspired many science fiction stories. The issues are quite real; we have no solid theory which includes time in a satisfying manner, along with quantum mechanics. I spent a great deal of space in my novel *Timescape* wrestling with how to make this intuitively clear, but the struggle to think in four dimensions is perhaps beyond realistic fiction; perhaps it is more properly the ground of metaphor.

Physicists began envisioning higher dimensions because they got a simpler dynamic picture, at the price of apparent complication. More dimensions to deal with certainly strains the imagination, and is at first glance an unintuitive way to think. But they can lead to beauties which only a mathematician can love, abstruse elegances. Thus Einstein, in his 1916 theory of general relativity, invoked the simplicity that objects move in "geodesics"–undisturbed paths, the equivalent of a straight line in Euclidean, rectangular geometry, or a great circle on a sphere–in a four-dimensional spacetime. The clarity of a single type of curve, in return for the complication of a higher dimension.

Einstein's general relativity said that matter curved the four-dimensional spacetime, an effect we see as gravity. Thus he replaced a classical idea, force, with a modern geometrical view, curvature of a 4D world. This led to a cosmology of the entire universe which was expanding, and therefore pointed implicitly backward to an origin.

Einstein did not in fact like this feature of his theory, and in his first investigations of his own marvelously beautiful equations fixed up the solution until it was static, without beginning or end. His authority was so profound that his bias might have held for ages, but Edmund Hubble showed within a decade that the universe was expanding.

Even so, the concept of a beginning (and perhaps an end) may be an artifact of our persistent 3D views. Implicitly, space and time separate in the Einstein universe. They are connected, but can be defined as ideas that stand alone.

The essence of talking about dimensions is that they can be separately described. But this may not be so. At least, not in the beginning.

Even Edwin Abbott did not foretell that in the hands of cosmologists like Stephen Hawking and James Hartle, time and space would blend. Though the universe remains 4D, definitions blur.

Following the universe back to its origins leads inevitably to an early instant when intense energies led to the breakdown of the very ideas of space and time. Quantum mechanics tells us that as we proceed to earlier and earlier instants, something peculiar begins to happen. Time begins to turn into space. The origin of everything is in spacetime, and the "quantum foam" of that primordial event is not separable into our familiar distances and seconds.

What is the shape of this spacetime? Theory permits a promiscuously infinite choice. Our usual view would be that space is one set of coordinates, and time another. But quantum uncertainty erupts through these intuitive definitions.

Begin with an image of a remorselessly shrinking space governed by a backward marching time, like a cone racing downward to a sharp point. Time is the length along the axis, space the circular area of a side-wise slice. Customarily, we think of the apex as the beginning of things, where time starts and space is of zero extent.

Now round off the cone's apex to a curve. There, length and duration smear. This rounded end permits no special time when things began. To see this imagine the cone tilted. This model universe could be conceptually tilted this way or that, with no unique inclination of the cone seems preferred. Now the "earliest" event is not at the center of the rounded end. It is some spot elsewhere on the rounded nub, a place where space and time blend. No particular spot is special.

Another way to say this is that in 4D, time and space emerge gradually from an earlier essence for which we have no name. They are ideas we now find quite handy, but they were not forever fundamental.

In the primordial Big Bang, there is no clear boundary between space and time. Rather than an image of an explosion, perhaps we should call this event the Great Emergence. There we are outside the conceptual space of precisely known space and well defined time. Yet there are still only four dimensions–just not sharp ones.

Einstein's cosmology thus begins with a time that is limited in the past, but has no boundary as such. Neither does space. As Stephen Hawking remarked, "The boundary condition of the universe is that it has no boundary."

Perhaps Edwin Abbott would not like the theological ramifications of these ideas. He was of the straitlaced Church of England. The American version is the Episcopal faith, which happens to be my own. (As a boy I was an acolyte, charged with light candles and carrying forth the sacraments of holy communion, in red and white robes. The robes were intolerably hot in our Atlanta church, and once I fainted and collapses in service–overcome by the heat, not the ideas. I'm told it provoked a stir.)

No doubt, psychologically the sharp-cone picture with its initial singular point suggests the idea of a unique Creator who sets the whole thing going. How? Physics has no mechanism. For now, it merely describes. There is a conceptual gap here, for we have no model which tells us a mechanism for making universes, much less one in which such basics as space and time are illusions. We need a "God of the gaps" to explain how the original, defining event happened. These new theories seem to bridge this gap in a fashion, but at the price of abandoning still more of our basic intuitions.

Much of God's essence comes from our perceived necessity for a creator, since there was a creation. But if there is no sharp beginning, perhaps we need no sharp, clear creator. Without a singular origin in time, or in space for that matter, is there any need to appeal to a supernatural act of creation?

But does this mean we can regard the universe as entirely self-consistent, its 4D nature emerging with time, from an event which lies a finite time in our past but does not need any sort of infinite Creator? Can the universe be a closed system, containing the reason for its very existence within itself?

Perhaps–to put it mildly. Theory stands mute. Yet this latest outcome of our wrestling with dimensions assumes that there are laws to this universe, mathematically expressed in a stew of coordinates and algebra and natural beauties.

But whence come the laws themselves? Is that where a Creator resides making not merely spacetime but the laws? Of this mathematics can say nothing–so far.

Edwin Abbott would no doubt be astonished at the twists and turns his Lewis Carroll-like narrative has taken us to, only a bit more than a century beyond his initial penning of Flatland. The questions still loom large.

So much matters progress, sharpening the questions without answering them in final fashion. We can only be sure that the future holds ideas which he, and we, would find stranger still.