Basil Blackwell, 1926.
Barnes and Nobel, 1963 through present.

Introduction by **William Garnett**

In an address to the Committee of the Cayley Portrait Fund in 1874 Clerk Maxwell, after referring in humorous terms to the work of Arthur Cayley in higher algebra and algebraical geometry, concluded his eulogium with the lines

March on, symbolic host! with step sublime,
Up to the flaming bounds of Space and Time!
There pause, until by Dickenson depicted,
In two dimensions, we the form may trace
Of him whose soul, too large for vulgar space
In n dimensions flourished unrestricted.

In those days any conception of "dimensions" beyond length, breadth and height was confined to advanced mathematicians; and even among them, with very few exceptions, the fourth and higher dimensions afforded only a field for the practice of algebraical analysis with four or more variables instead of the three which sufficiently describe the space to which our foot-rules are applicable. Any geometrical conclusions reached were regarded only as analogies to the corresponding results in geometry of three dimensions and not as having any bearing on the system of Nature. As an illustration, reference may be made to the "more divine offspring of the divine Cube in the Land of Four Dimensions" mentioned on p. 94 *infra* which has for its faces eight three-dimensional cubes and possesses sixteen four-dimensional angular points or corners.

During the present century the work of Einstein, Lorentz, Larmor, Whitehead and others has shewn that at least four dimensions of *space-time* are necessary to account for the observed phenomena of nature, and there are some suggestions of the necessity for more than four. It is only when dealing with very high velocities, such as are comparable with the velocity of light, that the unity of time with space thrusts itself upon the notice of physicists, for even with such a velocity as that of the planet Mercury in its orbit it is only after the lapse of centuries that any divergence from the motion strictly calculated on the basis of Euclidean Geometry and Newton's laws of gravitation and of motion has become apparent. The observed behaviour of electrons, moving in high vacua with velocities comparable with the velocity of light, has confirmed some of Einstein's conclusions and necessitated a revision of our fundamental notions of kinematics and the laws of motion when these high velocities are concerned. But the whole subject of Relativity has strongly appealed to popular interest through the brilliant confirmation of Einstein's theory of gravitation by the bending of light in passing close to the sun's surface and the consequent apparent displacement of stars which are very close to the sun from their true relative position when photographed during a solar eclipse. The best popular exposition of the whole subject of relativity and gravitation is to be found in Professor Eddington's *Space, Time, and Gravitation.*

But when a great truth comes to light it is generally found that there have already been prophets crying in the wilderness and preparing the way for the reception of the Revelation when the full time has come. In an anonymous letter published in *Nature* on February 12th, 1920, entitled "Euclid, Newton, and Einstein," attention was called to such a prophet in the following words:–

"Some thirty or more years ago, a little *jeu d'esprit* was written by Dr. Edwin Abbott, entitled 'Flatland.' At the time of its publication it did not attract as much attention as it deserved. Dr. Abbott pictures intelligent beings whose whole experience is confined to a plane, or other space of two dimensions, who have no faculties by which they can become conscious of anything outside that space and no means of moving off the surface on which they live. He then asks the reader, who has the consciousness of the third dimension, to imagine a sphere descending upon the plane of Flatland and passing through it. How will the inhabitants regard this phenomenon? They will not see the approaching sphere and will have no conception of its solidity. They will only be conscious of the circle in which it cuts their plane. This circle, at first a point, will gradually increase in diameter, driving the inhabitants of Flatlands outwards from its circumference, and this will go on until half the sphere has passed through the plane, when the circle will gradually contract to a point and then vanish, leaving the Flatlanders in undisturbed possession of their country. ... Their experience will be that of a circular obstacle gradually expanding or growing, and then contracting, and they will attribute *growth in time* what the external observer in three dimensions assigns to motion in the third dimension. Transfer this analogy to a movement of the fourth dimension through three-dimensional space. Assume the past and future of the universe to be all depicted in four-dimensional space and visible to any being who has consciousness of the fourth dimension. If there is motion of our three-dimensional space relative to the fourth dimension, all the changes we experience and assign to the flow of time will be due simply to this movement, the whole of the future as well as the past always existing in the fourth dimension."

It will be noticed that in the presentation of the Sphere to the Flatlander the third dimension involves time through the motion of the Sphere. In the Space-Time Continuum of the Theory of Relativity the fourth dimension is a time function, and the simplest element is an "event." One set of parallel sections of the four-dimensional continuum present the universe as it exists in three-dimensional space at the instants corresponding to the sections. Sections in all other directions involve the time element and represent the universe as it appears to an observer in motion.

There are some mathematical minds which are completely satisfied by the results expressed in algebraical symbols of the analysis of a continuum of four dimensions; but there are others which crave for the visualization of these results which, in their symbolic form, they do not question. To many, perhaps to the great majority, of these, Dr. Abbott's sphere penetrating Flatland points the way to the clearest imagery of the fourth dimension to which they are likely to attain.