

# THE STABILIZER OF IMMANANTS

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ABSTRACT. Immanants are homogeneous polynomials of degree  $n$  in  $n^2$  variables associated to the irreducible representations of the symmetric group  $\mathfrak{S}_n$  of  $n$  elements. We describe immanants as trivial  $\mathfrak{S}_n$  modules and show that any homogeneous polynomial of degree  $n$  on the space of  $n \times n$  matrices preserved up to scalar by left and right action by diagonal matrices and conjugation by permutation matrices is a linear combination of immanants. Building on works of M. Antónia Duffner [3] and Coelho, M. Purificação [1], we prove that for  $n \geq 6$  the identity component of the stabilizer of any immanant (except determinant, permanent, and  $\pi = (4, 1, 1, 1)$ ) is  $\Delta(\mathfrak{S}_n) \rtimes T(GL_n \times GL_n) \rtimes \mathbb{Z}_2$ , where  $T(GL_n \times GL_n)$  is the group consisting of pairs of  $n \times n$  diagonal matrices with the product of determinants 1, acting by left and right matrix multiplication,  $\Delta(\mathfrak{S}_n)$  is the diagonal of  $\mathfrak{S}_n \times \mathfrak{S}_n$ , acting by conjugation, ( $\mathfrak{S}_n$  is the group of symmetric group.) and  $\mathbb{Z}_2$  acts by sending a matrix to its transpose. Based on the work of Coelho, M. Purificação and Duffner, M. Antónia [2], we also prove that for  $n \geq 5$  the stabilizer of the immanant of any non-symmetric partition (except determinant and permanent) is  $\Delta(\mathfrak{S}_n) \rtimes T(GL_n \times GL_n) \rtimes \mathbb{Z}_2$ .

## REFERENCES

1. M. Purificação Coelho, *On linear preservers of immanants*, Linear Algebra Appl. **247** (1996), 265–271. MR 1412753 (97f:15011)
2. M. Purificação Coelho and M. Antónia Duffner, *On the conversion of an immanant into another*, Linear and Multilinear Algebra **44** (1998), no. 2, 111–130. MR 1674232 (99j:15009)
3. M. Antónia Duffner, *Linear transformations that preserve immanants*, Linear Algebra Appl. **197/198** (1994), 567–588, Second Conference of the International Linear Algebra Society (ILAS) (Lisbon, 1992). MR MR1275631 (95c:15012)

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