Final Exam
Math 102 Spring 2010

Instructions: This is a 3 hours long exam. You may not consult any notes or books during the exam, and no calculators are allowed. Show all of your work on each problem. Attach extra paper if you need more space.

Write your name:

Section (check one):

Section 001 Sevak Mkrtchyan MWF 9am	Section 002 Zheng Gan MWF 11am	Section 003 Elena Pavelescu MWF 11am

Write out the Honor Pledge: "On my honor, I have neither given nor received any unauthorized aid on this exam."

Signature:

Problem	Score
1	
2	
3	
4	
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10	
11	
12	
Total	

Some Taylor series centered at $x = 0$.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^k \frac{x^{2k+1}}{(2k+1)!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^k \frac{x^{2k}}{(2k)!} + \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots$$

Some trigonometric identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\sin A \cos B = \tfrac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$$

$$\sin A \sin B = \tfrac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\cos A \cos B = \tfrac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\int \tan x \; dx = \ln|\sec x| + C$$

$$\int \sec x \; dx = \ln|\sec x + \tan x| + C$$

1. Find the integral

$$\int \frac{2x-3}{(x+1)^2(x^2+4)}dx$$

2. Find the integral

$$I = \int \sin(x) e^x \, dx$$

3. Evaluate the definite integral

$$\int_0^4 \frac{\ln(x)}{\sqrt{x}}\,dx$$

4. Determine whether or not each of the following series converges. Note: before applying any test check (and show your work) that the hypotheses of the test are satisfied.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n^2}$

5. (a) Find the Maclaurin series of $\sin(2x)$.

(b) Use the previous part to find $\sin(1)$ within an error, $|error| < 10^{-3}$.

6. Compute the Taylor polynomial of degree 3 for $f(x) = x\ln(x)$ centered at $a = 1$.

7. (a) Find the sum of the series $\sum_{n=1}^{\infty} nx^{n-1}$.

(b) Use the previous part to evaluate $\sum_{n=1}^{\infty} \frac{n}{2^n}$

8. Consider the power series

$$\sum_{n=0}^{\infty} c_n(x-2)^n.$$

It is known that the power series converges when $x = 5$ and diverges when $x = -3$. For each of the following questions mark the correct answer (No credit will be given if it is ambigous which answer is chosen).

(a) The power series at $x = 4$

☐is convergent ☐is divergent ☐may or may not be convergent.

(b) The power series at $x = -1$

☐is convergent ☐is divergent ☐may or may not be convergent.

(c) Could the radius of convergence R be

i. $R = 2$? ☐Yes ☐No

ii. $R = 7$? ☐Yes ☐No

iii. $R = 0$? ☐Yes ☐No

iv. $R = \infty$? ☐Yes ☐No

v. $R = 4$? ☐Yes ☐No

vi. $R = 3$? ☐Yes ☐No

9. Find the length of the curve $y = \ln(\cos(x))$ when $0 \leq x \leq \frac{\pi}{3}$.

10. Find those values of t where the curve

$$\begin{aligned} x(t) &= t^3 - 3t \\ y(t) &= e^{2t} - e^t \end{aligned}$$

has

(a) a horizontal tangent.

(b) a vertical tangent.

11. Consider the curve which in polar coordinates is given by $r = 2 + \sin(3\theta)$.

(a) i. Find the range of θ for which r is increasing.

ii. Find the range of θ for which r is decreasing.

iii. Find the values of θ for which r is maximum.

iv. Find the values of θ for which r is minimum.

(b) Use the information from the previous part to plot the curve.

12. Find the area of the region inside both the cardioid $r = 1+\sin(\theta)$ and the circle $r = 3\sin(\theta)$.