Final Exam
Math 102 – Spring 2009

Instructions: This is a 3 hour exam. You may not consult any notes or books during the exam, and no calculators are allowed. Show all of your work on each problem. Attach extra paper if you need more space.

Name:

Honor Pledge: On my honor, I have neither received nor given any unauthorized aid on this exam.

Signature:

Problem	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total	

- **Problem 1**
  Compute the integral
$$\int \frac{x^2}{\sqrt{16-x^2}}dx$$

- **Problem 2**
  Compute the integral
$$\int \frac{x^2}{(x+2)^3}dx$$

- **Problem 3**
  Decide if the following improper integral is convergent or divergent. If it converges, give the value to which it converges.

$$\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

- **Problem 4**
  Find the third degree Taylor polynomial of $f(x) = \sqrt{x}$, at $a = 1$.

- **Problem 5**

A. Consider the function

$$f(x) = \frac{\ln x}{2x}, \text{ for } x \geq 3$$

Is $f(x)$ increasing, decreasing, or neither on the interval $[3, \infty)$? Justify your answer.

B. Is the following series convergent or divergent? Justify your answer.

$$\sum_{n=3}^{\infty} \frac{(-1)^n \ln n}{2n}$$

C. Is the series from B absolutely convergent? Justify your answer.

- **Problem 6**
  Decide if the following series are convergent or divergent? Be sure to show that you've checked the hypotheses for any tests you use.

  A.

$$\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$$

  B.

$$\sum_{n=1}^{\infty} n \cdot \sin \frac{1}{n}$$

C.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 + n^3}$$

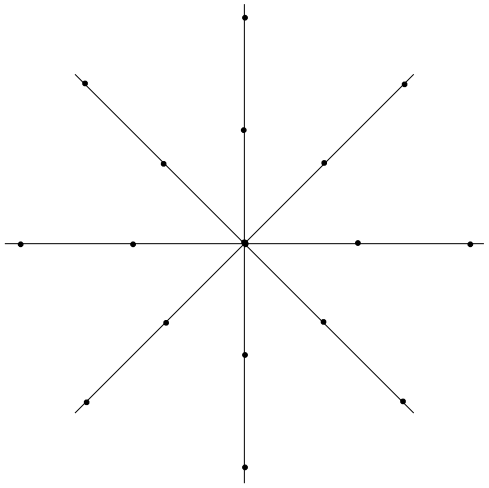
- **Problem 7**
  Find the interval of convergence for the Taylor series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{3n-1}.$$

  Don't forget to check the endpoints of the interval and to show that you've check the hypotheses for any tests you use.

- **Problem 8**
  Let $f(\theta) = 2\sin(2\theta)$.

  A. Sketch the graph of the polar equation $r = f(\theta)$.



  B. Calculate the area of the region enclosed by the four loops of the graph of $r = f(\theta)$ which also lies inside the circle $r = 1$.

- **Problem 9**
  Consider the point A given in rectangular coordinates by $(x, y) = (3, -3\sqrt{3})$. Find the polar coordinates (r, θ) of A, where $r > 0$ and $0 \leq \theta < 2\pi$.

- **Problem 10**
  Consider the parametric curve $\mathcal{C}$ given by $x(t) = e^t$, $y(t) = t^2$.

  A. What is the slope of the line tangent to $\mathcal{C}$ at the point $(x(1), y(1))$?

  B. Find the area between $\mathcal{C}$ and the x-axis for $0 \leq t \leq 1$.

- **Problem 11**
  Consider the parametric curve $\mathcal{C}$ given by $x(t) = 2\sin t$, $y(t) = -2\cos t$, $0 \leq t \leq \pi$.

  A. Graph this curve and specify its direction as the parameter t goes from 0 to π.

  B. Compute the length of the curve $\mathcal{C}$.

Half and double angle identities

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

Integrals of basic trigonometric functions

$$\int \tan x\,dx = -\ln|\cos x| + C = \ln|\sec x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C = -\ln|cscx| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$