1. Find the interval of convergence of the following infinite series. Be sure to test the endpoints of the interval.

$$\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$$

2. (a) Find the sum of the series $\sum_{n=1}^{\infty} nx^{n-1}$.

   (b) Use the previous part to evaluate $\sum_{n=1}^{\infty} \frac{n}{2^n}$

3. Consider the power series

$$\sum_{n=0}^{\infty} c_n(x-2)^n.$$

   It is known that the power series converges when $x = 5$ and diverges when $x = -3$. Is it possible the radius of convergence R is

   (a) $R = 2$? ☐Yes ☐No

   (b) $R = 4$? ☐Yes ☐No

   (c) $R = 5$? ☐Yes ☐No

4. Find the Taylor polynomial of degree 3 for $f(x) = \tan^{-1} x$ at $a = 0$. Use this polynomial to approximate $\frac{\pi}{6}$.

5. Use the Taylor series of $\cos x$ to find a power series representation of $f(x) = x \cdot \cos(2x)$.

ANSWERS:

1. $[-1, 1)$
2. $\frac{1}{(1-x)^2}$; 2
3. No. Yes. Yes.
4. $T_3(x) = x - \frac{1}{3}x^3$. $\frac{\pi}{6} = \tan^{-1}(\frac{\sqrt{3}}{3}) \sim \frac{8\sqrt{3}}{27}$
5. $$\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n+1}}{(2n)!}$$