

1. Find a power series representation of the function $f(x) = \frac{x}{(1-2x)^2}$. Determine the radius of convergence of this series.
2. Find the second Taylor polynomial of $f(x) = \sqrt{x+1}$ at $a = 0$. Use this polynomial to approximate $\sqrt{0.9}$.
3. Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (3x)^n}{n^2}$$

Are there any values of x for which the series is conditionally convergent?

4. Find a power series representation of $g(x) = \frac{\sin(x^2) - x^2}{x^3}$ centered at $a = 0$. What is the radius of convergence of the series?
5. Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{n!}$$

ANSWERS:

1.

$$\sum_{n=1}^{\infty} 2^{n-1} n x^n$$

$$R = \frac{1}{2}$$

2. $T_2(x) = 1 + \frac{x}{2} - \frac{x^2}{8}$; $\sqrt{0.9} \sim \frac{759}{800}$

3. $[-\frac{1}{3}, \frac{1}{3}]$. No.

4.

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^{4n-1}}{(2n+1)!}$$

$$R = +\infty$$

5. $e^{-4} - 1$
