Pledge: On my honor I have neither given nor received any aid on this exam.

Signature:

Printed name:

This is a closed notes, closed book, no calculator exam. You have 50 minutes to complete the exam. Remember to sign the pledge above when you finish the exam.

Please work carefully. Show your work, write your answers clearly, simplify your answers as much as possible, and double check your test!! Justify all of your work. Make sure to ask if you don't understand a question. Attach additional paper if necessary.

Helpful hint: Do the problems you find easy first!!!

20 points each

1. ________
2. ________
3. ________
4. ________
5. ________
6. ________
7. ________
8. ________

total ________

1. *(10 points)* Find the volume of the solid in the first octant bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $z + 2x + 2y = 2$.

2. *(15 points)*

a. Compute

$$\int_C (\sin z)dx + (\cos z)dy - (xy)^{1/3}dz,$$

where C is the path parametrized by $\bar{c}(t) = (\cos^3 t, \sin^3 t, t)$ for $0 \le t \le 3\pi/2$.

b. Suppose $\overline{F} = P\vec{i} + Q\vec{j} = (1 + \tan x)\vec{i} + (x^2 + e^y)\vec{j}$, and C is the boundary of the region (oriented counterclockwise) contained within the curves $y = \sqrt{x}$, $x = 1$ and $y = 0$. Compute

$$\int_C P dx + Q dy.$$

3. (*10 points*) Let D be a rectangle with corners $(0,0),(1,2),(2,1)$ and $(1,-1)$. Compute

$$\iint_D (x-2y)\,dA$$

using the change of variables $x=u+v$, $y=2u-v$.

4. *(10 points)*

   a. Let S denote the surface $x^6 + y^6 + z^6 = 1$ (this is sort of like a cube with rounded corners). Compute

   $$\int\int_S \overline{F}.d\bar{S}$$

   where S is oriented with outward pointed normal and $\overline{F}$ is the vector field $\mathbf{F} = e^z\vec{i} + x\vec{j} + \tan x\vec{k}$.

   b. Let S denote the sphere $x^2 + y^2 + z^2 = 6$ with unit normal $\vec{n}$ oriented outwards, and let $\overline{F}$ denote the vector field $\overline{F} = z\vec{i} - 2y\vec{j} + xy\vec{k}$. Compute

   $$\int\int_S \overline{F}.d\bar{S}$$

5. *(10 points)* Let C denote the closed rectangular path from $(0,0,0)$ to $(1,0,1)$ to $(1,1,1)$ to $(0,1,0)$ back to $(0,0,0)$. (This path lies within a plane). Compute

$$\int_C \overline{F}.d\bar{s}$$

where $\overline{F} = x^2\vec{j}$.

Take home portion of exam 2
Due Monday, April 14, 11:00 AM

Options for turning it in:

1. Turn it in IN CLASS on Monday at 11:00 AM.
2. Put it under my door by 11:00 AM Monday (Herman Brown 212).

Pledge: On my honor I have neither given nor received any aid on this exam.

Signature:

Printed name:

Date: **Start time:** **Stop time:**

This is a closed notes, closed book, no calculator exam. It should just be you, a pencil or pen and the exam. You have **1 hour** to complete the exam, and you must do so in one sitting. Remember to sign the pledge above when you finish the exam. **Do not open this exam until you are ready to sit down and take the exam.**
Please work carefully. Show your work, write your answers clearly, simplify your answers as much as possible, and double check your test!! Justify all of your work. Make sure to ask if you don't understand a question. Attach additional paper if necessary.
Helpful hint: Do the problems you find easy first!!!

6. *(15 points)*

a. Constant heavy rain falls straight down, described by the vector field $\overline{F} = -3\vec{k}$. Find the total flux through the cone $z = (x^2 + y^2)^{1/2}$, $x^2 + y^2 \leq 1$ which is oriented with normal vectors pointing in the negative z direction.

b. Let S be the cone given in part (a). Compute

$$\int\int_S (\nabla \times \overline{F}).d\bar{S}$$

where $\overline{F} = (-y, x, xe^x yz)$.

7. *(15 points)*

a. Give a parametrization ($\Phi(u, v)$) for the ellipsoid with equation

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 1.$$

b. Calculate T_u, T_v and $T_u \times T_v$ for your parametrization.

c. Draw the ellipsoid from part (a) and indicate, using a normal vector, the orientation given by **your** parametrization.

d. Using parts (a) and (b) write down an integral whose value is the surface area of the ellipsoid from part (a). You do not have to evaluate this integral.

8. *(15 points)* Consider the following two vector fields in $\mathbb{R}^3$.

(i) $\overline{F} = (x^3 - 3xy^2)\vec{i} + (y^3 - 3x^2y)\vec{j} + z\vec{k}$

(ii) $\overline{G} = y^2\vec{i} - z^2\vec{j} + x^2\vec{k}$.

a. Which of these vector fields (if any) are conservative?

b. Find potential functions for the vector fields that are conservative.

c. Let $\bar{p}$ be the path that goes from $(0,0,0)$ to $(2,2,2)$ by following the edges of the cube $0 \leq x \leq 2$, $0 \leq y \leq 2$, $0 \leq z \leq 2$ from $(0,0,0)$ to $(0,0,2)$ to $(0,2,2)$ to $(2,2,2)$. Let $\bar{c}$ be the path from $(0,0,0)$ to $(2,2,2)$ directly along the diagonal of the cube. Find the values of the line integrals

(i) $\int_{\bar{p}} \overline{F}.d\bar{s}$

(ii) $\int_{\bar{p}} \overline{G}.d\bar{s}$

(iii) $\int_{\bar{c}} \overline{F}.d\bar{s}$

(iv) $\int_{\bar{c}} \overline{G}.d\bar{s}$