

1)a) Let $v = v_1e_1 + \dots + v_ne_n \in V \simeq \mathbb{R}^n$ and

$$\omega = e^1 \wedge e^2 \wedge e^3 + e^1 \wedge e^3 \wedge e^4 \in \bigwedge(V^*).$$

Compute the interior multiplication $i(v)\omega$.

b) Describe the kernel of $i(e_1)$ in $\bigwedge_3(V^*)$.

2) Let V be a real vector space of dimension n .

a) Show that if $\omega \in \bigwedge_2(V)$ is decomposable then $\omega \wedge \omega = 0$.

b) If $n = 4$ show the converse holds.

3) Consider vectors $v_1, \dots, v_k \in V$, a real vector space. Show that

$$v_1 \wedge \dots \wedge v_k \neq 0$$

if and only if $\{v_1, \dots, v_k\}$ is linearly independent.