

(Math 180, Midterm (2), Spring 2009 April 2, 5-7pm) Do all of your work in the bluebook and show all your computations. You need to justify all your answers.

Problem 1.(20pt) Find the limit if exists, or show that the limit does not exist.

(1) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy \cos y}{\sqrt{x^2+y^2}}$

(2) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + \sin^2 y}{2x^2 + 3y^2}$

Problem 2.(10 pt) Find the points on the surface $x^2 + 4y^2 - z^2 = 2x$ where the tangent plane is parallel to the plane $z = -2x - 2y + 3$.

Problem 3.(10 pt) The length x of a side of a triangle is increasing at a rate of 2 in /s, the length y of another side of a triangle is decreasing at a rate of 3 in /s, and the angle θ between the two sides is increasing at a rate of 2 radian /s. How fast is the area of the triangle changing when $x = 4$ in, $y = 2$ in, and $\theta = \frac{\pi}{6}$.

Problem 4.(10 pt) A plane is flying above the mountains with unit speed and with constant altitude with respect to the sea level. If the plane is travelling northeast while passing through a point P , then the rate of change of its altitude from the ground is $\sqrt{2}$. If it is traveling northwest while passing through P , then the rate of change of its altitude from the ground is $-2\sqrt{2}$. What is the rate of change of the plane's altitude from the ground when it is travelling south?

Problem 5 . (10 pt) Find the absolute maximum and the minimum values of $f(x, y) = e^{-x^2-y^2}(x^2 + 2y^2)$ on the disk $D : x^2 + y^2 \leq 4$.

Problem 6.(10pt) Consider the curve in the plane defined by the equation $\sin(x-y) = \frac{2}{\pi}xe^y$. Find the slope of the tangent line of this curve at $P = (\pi/2, 0)$.

Problem 7.(20pt) The base of aquarium with a given volume V is made of slate and the sides are made of glass, If slate costs ten times as much (per unit area) as glass, find the dimensions of the aquarium that minimize the cost of the material.

Problem 8.(10pt) Find the volume of the solid enclosed by the surface $z = 2 + e^x \sin y$ and the planes $x = \pm 2$, $y = 0$, $y = \pi$ and $z = 0$.