# Errata and Corrections to *The Arithmetic of Dynamical Systems* 1st Edition

Joseph H. Silverman

May 8, 2014

**Acknowledgements Page vi**
I would like to thank the following people for sending me comments and corrections: Jackie Anderson, Rob Benedetto, Nils Bruin, John Cullinan, John Doyle, Xander Faber, Anthony Flatters, Liang-Chung Hsia, Trevor Hyde, Patrick Ingram, Chong Gyu Lee, Alex Molnar, Giulio Peruginelli, Bob Rumely, Nantawat Udomchatpitak, Kwon Young-woo.

**Page 4, line 25**
The sentence "In particular, we prove that $\mathcal{M}_2$ is an isomorphism to the affine plane $\mathbb{A}^2$." should read "In particular, we prove that $\mathcal{M}_2$ is isomorphic to the affine plane $\mathbb{A}^2$."

**Page 4, line** $-6$
The book says that $\mathbb{P}^1(\mathbb{C}_p)$ is not Hausdorff, but it is clear that $\mathbb{P}^1(\mathbb{C}_p)$ is a Hausdorff space.

**Page 9, Line** $-1$ **and Page 63, Line** $8$
There is inconsistant notation in labeling the coefficients of rational functions. Page 9 gives a rational function as

$$\phi(z) = \frac{F(z)}{G(z)} = \frac{a_0 + a_1 z + \cdots + a_d z^d}{b_0 + b_1 z + \cdots + b_d z^d}$$

and page 63 gives a rational function as

$$\phi(z) = \frac{a_0 z^d + a_1 z^{d-1} + \cdots + a_{d-1} z + a_d}{b_0 z^d + b_1 z^{d-1} + \cdots + b_{d-1} z + b_d}$$

This should be made consistant throughout the book.

**Page 41, Exercise 1.30(f)**
Replace $n$ in the binomial coefficient with $d$.

**Page 48, line** $-1$
$\mathbb{P}^1(k)$ should be $\mathbb{P}^N(k)$

**Page 49, line** 2
$\mathbb{P}^1(k)$ should be $\mathbb{P}^N(k)$

**Page 49, line** 1 **of example 2.6**
$\mathbb{P}^1(\mathbb{Q})$ should be $\mathbb{P}^3(\mathbb{Q})$

**Page 51, Proof of Proposition 2.11**
The inqualities in the first three lines of the proof should be reversed. Thus it should read

Write $P_i = [X_i, Y_i]$ with normalized coordinates. If $v(X_1) < v(Y_1)$, we begin by applying the map $f = Y/X \in \mathrm{PGL}_2(R)$ to each of the three points, so we may assume that $v(X_1) \geq v(Y_1)$. Since the coordinates are normalized, this implies that $v(Y_1) = 0$, so $Y_1$ is a unit.

**Page 62, line** 1
The penultimate $\phi$ on this line should have a tilde. Thus it should read

$$\tilde{P} = \tilde{\phi}^n(\tilde{P}) = \tilde{\phi}^r \circ \underbrace{\tilde{\phi}^m \circ \cdots \circ \tilde{\phi}^m}_{k \text{ iterations}}(\tilde{P}) = \tilde{\phi}^r(\tilde{P}).$$

**Page 79, Exercise 2.24**
Add the assumption that the residue characteristic is not equal to 2.

**Page 84, line -9**
There is a missing absolute value symbol in the middle part of the displayed formula. It should read

$$\prod_{v \in M_K} \max_i \{|\alpha x_i|_v\}^{n_v} = \prod_{v \in M_K} |\alpha|_v^{n_v} \max_i \{|x_i|_v\}^{n_v} = \prod_{v \in M_K} \max_i \{|x_i|_v\}^{n_v}.$$

**Page 85, Second displayed formula**
$H(P)$ should be $H_{\mathbb{Q}}(P)$, since $H(P)$ hasn't been defined yet. So this line should read

$$\big\{P \in \mathbb{P}^N(\mathbb{Q}) : H_{\mathbb{Q}}(P) \leq B\big\}$$

**Page 86, Third line of first displayed formula**
There's a missing $n_v$ in the exponent. So this full display should read

$$\begin{aligned} H_L(\sigma(P)) &= \prod_{v\in M_L} \max\{|\sigma(x_0)|_v,\dots,|\sigma(x_N)|_v\}^{n_v} \\ &= \prod_{v\in M_L} \max\left\{|x_0|_{\sigma(v)},\dots,|x_N|_{\sigma(v)}\right\}^{n_{\sigma(v)}} \\ &= \prod_{v\in M_L} \max\{|x_0|_v,\dots,|x_N|_v\}^{n_v} \\ &= H_L(P). \end{aligned}$$

**Page 92, bottom**
The coefficients of the polynomials $g_{ij}$ whose existence comes from the Nullstellensatz lie in $\bar{K}$, but the subsequent proof suggests that the $g_{ij}$ have coefficients in $K$. There are two ways around this apparent problem. The easiest is to simply replace $K$ with the finite extension field obtained by adjoining all of the coefficients of all of the $g_{ij}$'s. The compatibility of the height under field extensions says that we can do this. Alternatively, it is not hard (using some elementary algebraic number theory) to show that in fact one can choose the $g_{ij}$ to have coefficients in $K$.

**Page 102, First displayed formula**
This is supposed to be the absolute height, so there should be a factor of $1/[K:\mathbb{Q}]$. Thus this line should read

$$h(P) = h(\alpha) = \frac{1}{[K:\mathbb{Q}]} \sum_{v\in M_K} n_v \log \max\{|\alpha|_v, 1\},$$

**Page 103, Theorem 3.29, Equation (3.17)**
This is supposed to be the absolute height, so there should be a factor of $1/[K:\mathbb{Q}]$. Thus this line should read

$$\hat{h}_\phi(\alpha) = \frac{1}{[K:\mathbb{Q}]} \sum_{v\in M_K} n_v \hat{\lambda}_{\phi,v}(\alpha) \qquad \text{for all } \alpha \in \mathbb{P}^1(K) \smallsetminus \{\infty\}.$$

**Page 105, Theorem 3.35**
The case $B = 0$ must be excluded.

**Page 105, Proof of Theorem 3.35**
Need to explain why the plane curves $G_1(X,Y) = B_1$ and $G_2(X,Y) = B_2$ have no common components.

**Page 108, Theorem 3.40 (Roth's theorem)**
Need to specify that $\alpha_v \notin K$, so the statement of the theorem should read "choose an algebraic number $\alpha_v \in \bar{K} \smallsetminus K$."

**Page 111, 2nd displayed equation**
The variable should be $z$, not $x$. Thus

$$\phi(z) = \frac{899z^2 - 2002z + 275}{33z^2 - 584z + 275}.$$

**Page 111, Proposition 3.46**
The rational map is supposed to have degree $d$. So it should read:

**Proposition 3.46.** *For all integers $N \geq 0$ and $d \geq 2$ there exists a rational map $\phi(z) \in \mathbb{Q}(z)$ of degree $d$ with the following properties*:

**Page 113, Table 3.1, Caption**
The caption should be $\mathcal{O}_\phi(1)$, not $\mathcal{O}_1(\phi)$.

**Page 114, Line** $-8$
It is not true that

$$\frac{1}{|a_n|^\epsilon} \geq \left|\frac{b_n}{a_n}\right|$$

is a consequence of $|a_n| \geq |b_n|^{1+\epsilon}$. So this needs to be adjusted. What is true is that

$$|a_n| \geq |b_n|^{1+\epsilon} \Longrightarrow \frac{1}{|a_n|^{\epsilon/(1+\epsilon)}} \geq \left|\frac{b_n}{a_n}\right|.$$

We may assume that $\epsilon \leq \frac{1}{2}$, which implies that

$$\frac{\epsilon}{1+\epsilon} \geq \frac{2}{3}\epsilon.$$

So assuming $\epsilon \leq \frac{1}{2}$, we have

$$|a_n| \geq |b_n|^{1+\epsilon} \Longrightarrow \frac{1}{|a_n|^{2\epsilon/3}} \geq \left|\frac{b_n}{a_n}\right|.$$

So can change the exponent on line $-8$ on page 114 to $\frac{2}{3}\epsilon$, and then line 3 on page 115 needs to be changed to

$$\log|a_n| \leq \frac{6}{\epsilon}\log(C_5^{-1}).$$

**Page 115, Line** 1
It should be $\geq$ instead of $=$.

**Page 115, Line** 3
It should be $C_5^{-1}$ instead of $C_5$, so the inequality should read

$$\log|a_n| \leq \frac{2}{\epsilon}\log(C_5^{-1}).$$

(However, see also the correction listed above for page 114, line −8.)

**Page 118, Equation (3.33)**
The "$f$" should be "$\phi$". Thus it should read

$$e_P(\phi^2) = e_P(\phi)e_{\phi(P)}(\phi) = e_0e_1 \le d^2 - d.$$

**Page 120, Line 3 (Second displayed equation)**
In the definition of $N(\phi, \alpha, \epsilon, \beta)$, the $\beta_n$ should just be $\beta$. Thus this line should read

$$N(\phi, \alpha, \epsilon, \beta) = \Big\{ n \in N(\phi, \alpha, \epsilon) : \rho\big(\phi^{n-m}(\alpha), \beta\big)^{\mathbf{e}_\infty(\phi^m)} \le C_1 \rho\big(\phi^n(\alpha), \infty\big) \Big\},$$

**Page 121, First displayed equation**
The constants $C_7, C_8, C_9$ should be $C_8, C_9, C_{10}$, since $C_7$ was already used on the previous page. Thus the displayed equation should read

$$\begin{aligned}
\frac{1}{|a_n|^\epsilon} &\ge \rho(\phi^n(\alpha), \infty) && \text{from (3.34),} \\
&\ge C_8 \rho\big(\phi^{n-m}(\alpha), \beta\big)^{\mathbf{e}_\infty(\phi^m)} && \text{from (3.36),} \\
&= C_8 \left( \frac{b_{n-m}}{\sqrt{a_{n-m}^2 + b_{n-m}^2}} \right)^{\mathbf{e}_\infty(\phi^m)} && \text{definition of } \rho, \\
&\ge \frac{C_9}{H\big(\phi^{n-m}(\alpha)\big)^{\mathbf{e}_\infty(\phi^m)}} && \text{definition of height, where note that } b_{n-m} \ne 0, \text{ since } \alpha \text{ is wandering and } \infty \text{ is periodic,} \\
&\ge \frac{C_{10}}{H\big(\phi^n(\alpha)\big)^{\mathbf{e}_\infty(\phi^m)/d^m}} && \text{from Theorem 3.11,} \\
&\ge \frac{C_{10}}{H\big(\phi^n(\alpha)\big)^{\epsilon/6}} && \text{from (3.35),} \\
&= \frac{C_{10}}{|a_n|^{\epsilon/6}} && \text{since } H\big(\phi^n(\alpha)\big) = H(a_n/b_n) = |a_n|.
\end{aligned}$$

**Page 128, footnote**
Add the assumption that the boundary of $U$ has measure 0. So the footnote should read: "Recall that a sequence of measures $\mu_i$ on a compact space $X$ *converges weakly* to $\mu$ if for every Borel-measurable set $U$ with boundary satisfying $\mu(\partial U) = 0$, the sequence of values $\mu_i(U)$ converges to $\mu(U)$ as $i \to \infty$.

**Page 130, Line −9 before Proposition 3.63**
The point $P$ has not been defined. So replace the sentence starting "Choose a prime ideal $\mathfrak{P}\ldots$" with the following:

Let $P \in \mathrm{Per}_n^{**}(\phi)$ be a point of exact period $n$, and choose a prime ideal $\mathfrak{P}$ in $K_{\phi,n}^0(P)$ lying above $\mathfrak{p}$.

**Page 131, Corollary 3.64**
It is not clear why Proposition 3.63 implies that the points of formal period $n$ remain distinct when reduced modulo $\mathfrak{p}$ for the indicated primes. It is true that Proposition 3.63 says (more-or-less) that the point and its reduction have the same period, but why does that imply injectivity?

**Page 135, Exercise 3.3**
The quantity $D(P)$ is not defined. It is supposed to be the degree of the field of definition of $P$. So add the following at the beginning of this exercise:

For any $P \in \mathbb{P}^N(\bar{\mathbb{Q}})$, let $D(P) = [\mathbb{Q}(P) : \mathbb{Q}]$ be the degree of the field of definition of $P$.

**Page 137, Exercise 3.9**
Part (d) should certainly be marked as a hard problem, and possibly (c) should, too.

**Page 137, Exercise 3.13**
The stated result is true for any perfect field. For the first part of the proof, one assumes that $\phi^n(\sigma(P)) = \phi^m(P)$ for some $n \neq m$ and some $\sigma \in \mathrm{Gal}(\bar{K}/K)$ and shows that $P$ is preperiodic. Over number fields, I had in mind the following argument using the Galois invariance of the canonical height:

$$\begin{aligned}
\hat{h}\big(\phi^n(\sigma(P))\big) &= \hat{h}\big(\phi^m(P)\big),\\
d^n\hat{h}(\sigma(P)) &= d^m\hat{h}(P),\\
d^n\hat{h}(P) &= d^m\hat{h}(P),\\
\hat{h}(P) &= 0,
\end{aligned}$$

and hence $P$ is preperiodic. However, one can instead use a simple combinatorial argument and the fact that $\{\sigma^i P : i \geq 0\}$ is finite to prove this fact in general. Then the rest of the proof works over any perfect field.

**Page 138, Exercise 3.17**
Specify that the map $\phi$ has degree $d \geq 2$.

**Page 139, Exercise 3.21**
(b) The displayed equation

$$e^{n^2\hat{h}_\phi(\alpha)} - \phi^n(2)$$

should be

$$e^{2^n\hat{h}_\phi(\alpha)} - \phi^n(2).$$

(In other words, the exponent should have $2^n$, not $n^2$.)

(c) "is the closest integer to $H^{n^2}$" should be "is the closest integer to $H^{2^n}$"

**page 139, Exercise 3.22**
$H^{n^2}$ should be $H^{d^n}$.

**page 139, Exercise 3.23**
In the displayed equation, the exponent should have $d^n$ instead of $n^2$. Thus it should read

$$\left| e^{d^n \hat{h}_\phi(\alpha)} + \frac{a}{d} - \phi^n(\alpha) \right| \le \epsilon \qquad \text{for all } n \ge 0.$$

(Also move the $d^n$ to the other side of the $\hat{h}_\phi(\alpha)$, which is not an error, but makes the notation in Exercise 3.23 consistant with the notation in Exercise 3.21(b).)

**Page 140, Exercise 3.24(b)**
This is supposed to be the absolute height, so there should be a factor of $1/[K:\mathbb{Q}]$. Thus this line should read

$$\hat{h}_\phi(\alpha) = \frac{1}{[K:\mathbb{Q}]} \sum_{v \in M_K} n_v \hat{\lambda}_{\phi,v}(\alpha).$$

**Page 140, Exercise 3.32(a)**
The bound should be $\sqrt{4|B|/3}$ (missing absolute value sign).

**Page 144, Line 13**
Instead of $\alpha \in \mathbb{P}^N(\mathbb{Q}) \smallsetminus \mathrm{PrePer}(\phi)$, it should say $\alpha \in \mathbb{P}^1(\mathbb{Q}) \smallsetminus \mathrm{PrePer}(\phi)$.

**Page 146, Exercise 3.49(a)**
It should be "for some $e \in \mathbb{Q}$," not "for some $e \in \mathbb{Q}(c)$."

**Page 150, Definition**
Reverse the order of the 2nd and 3rd bullet items to match the order of $\mathrm{Per}_n$, $\mathrm{Per}_n^*$, and $\mathrm{Per}_n^{**}$.

**Page 151, Theorem 4.5(b) and proof**
Should not use $\lambda(P)$ to denote the multiplier at $P$, since that's not consistant with earlier notation. Use either $\lambda_P$ and $\lambda_P(\phi)$.

**Page 153, Line 2**
Insert "be" between "may" and "done".

**Page 158, Line −8**
Replace "primitive-$n$ periodic" with "primitive $n$-periodic". (Misplaced hyphen)

**Page 159, Line 12 (third displayed equation)**

$$\psi_t(z) = z^2 - (t + t^{-1} - 1)z \qquad \text{and} \qquad \lambda(t) = t - 1$$

The function should be $z^2 - \ldots$, not $z^2 + \ldots$. So the full line should read

**Page 159**
Should not use $\lambda$ to denote a map, since too easily confused with the use of $\lambda$ as the multiplier of a map.

**Page 160, equation (4.17)**
The righthand side should be $\Phi^*_{n,P}$, i.e., add a star.

**Page 161, first paragraph**
It is not clear, *a priori*, that $\phi$ induces an automorphism of exact order $n$ on $Y_0(n)$. One needs to know that there is at least one value of $c$ for which $\phi_c(z) = z^2 + c$ has a point of primitive period $n$. The description of the bifurcation polynomials in Section 4.2.4 and the identification of their roots with a finite set of points of the Mandelbrot set gives the desired result.

**Page 174, Bottom of page**
The action of $\mathrm{PGL}_2$ on $\mathbb{Q}[\mathrm{Rat}_d]$ is defined as

$$R^f(\phi) = R(\phi^f).$$

However, this is a *left action*, i.e., $R^{fg} = (R^g)^f$, so the notation is confusing. It might be better to write the action as ${}^fR$. Otherwise add a note indicating that it is a left action.

**Page 180, line 3**
It should be $\lambda_P(\phi) \in \mathbb{C}$, not $\lambda_P(\phi) \in \mathbb{C}^*$, i.e., the multiplier may be 0.

**Page 180, line −3**
It is true that $\lambda_{P_i}(\phi)$ is integral over $\mathbb{Q}[\mathrm{Rat}_d]$, but the ring listed here is not $\mathbb{Q}[\mathrm{Rat}_d]$. The correct description of $\mathbb{Q}[\mathrm{Rat}_d]$ in Proposition 4.27 on page 169. Thus

$$QQ[\mathrm{Rat}_d] = \mathbb{Q}\left[a_0, \ldots, b_d, \mathrm{Res}(F_{\mathbf{a}}, F_{\mathbf{b}})^{-1}\right]^{(0)},$$

where the superscript (0) denotes the elements of degree 0, i.e., quotients of homogeneous polynomials of the same degree.

**Page 187, lines 5 and 6**
Change $M_4$ to $\mathcal{M}_4$

**Page 187, Theorem 4.53**
The statement of McMullen's theorem is not quite what is given in [294]. One needs to do a little bit of algebraic geometry to pass from McMullen's result

to what is stated in Theorem 4.53. (I thank Xander Faber for pointing this out.)

**Page 187, lines 18 and 20**
Change $\boldsymbol{\sigma}_{d,N^*}$ to $\boldsymbol{\sigma}^*_{d,N}$

**Page 191, Proof of (a)**
Even assuming $\lambda_1\lambda_2 \neq 1$ and using (4.37), it does not follow that $\lambda_1 \neq 1$ and $\lambda_2 \neq 1$. However, all that we really neeed is that the fixed points associated to $\lambda_1$ and $\lambda_2$ are distinct. So replace the first two sentences of the proof of (a) with the following:

(a) For this part we assume that $\lambda_1\lambda_2 \neq 1$. If $\lambda_1 \neq \lambda_2$, then the fixed points associated to $\lambda_1$ and $\lambda_2$ are clearly distinct. But if $\lambda_1 = \lambda_2$, then our assumption implies that $\lambda_1$ and $\lambda_2$ are not equal to 1, so again the fixed points must be distinct. Hence we can find an element of $\mathrm{PGL}_2(\mathbb{C})$ that moves them to 0 and $\infty$, respectively.

**Page 192, Line** $-10$
The words "2-equivalent" should read "$\mathrm{PGL}_2$-equivalent." So the entire phrase reads:

Then Lemma 4.59(a)says that $\phi_1$ and $\phi_2$ are $\mathrm{PGL}_2$-equivalent to the function $(z^2 + \lambda_1 z)/(\lambda_2 z + 1)$,...

**Page 199, Line 15**
"and those that do, fall into finitely many..." Remove the comma after "do".

**Page 202, Definition**
The definition of cohomologous 1-cycles is not correct if $A$ is not abelian. The correct definition is that the 1-cocycles $g_1$ and $g_2$ are cohomologous if there is an $f \in A$ such that

$$g_{2,\sigma} = f g_{1,\sigma} \sigma(f^{-1}) \qquad \text{for all } \sigma \in G.$$

Note that if $A$ is abelian, then this is equivalent to $g_{1,\sigma}^{-1} g_{2,\sigma} = f\sigma(f^{-1})$ being a coboundary.

**Page 202, Remark 4.78**
Using the correct definition of cohomologous cycles, the indicated map is a well-defined injection of $\mathrm{Twist}(X/K)$ into $H^1(\mathrm{Gal}(\bar{K}/K), \mathrm{Aut}(X))$ even in the case that $\mathrm{Aut}(X)$ is nonabelian.

**Page 203, Theorem 4.79**
"Let $\phi(z) \in K(z)$ be a nonzero rational map..." should be "Let $\phi(z) \in K(z)$ be a nonconstant rational map..."

**Page 204, Remark 4.80**
It is true that $H^1\big(\mathrm{Gal}(\bar{K}/K), \mathrm{PGL}_2(\bar{K})\big)$ injects into $H^2\big(\mathrm{Gal}(\bar{K}/K), \bar{K}^*)[2]$,

but it is not a formal argument using exact sequences. In general, if $B$ is a non-abelian group on which $G$ acts, then the cohomology group $H^1(G,B)$ is a pointed set. Then for any exact sequence of groups on which $G$ acts $0 \to A \to B \to C \to 0$, assuming $A$ is in the center of $B$, we get an exact sequence of pointed sets

$$H^1(G,A) \to H^1(G,B) \to H^1(G,C) \to H^2(G,A).$$

However, an exact sequence of pointed sets only means that kernels equal images, and a kernel is the inverse image of the distinguished point. So for example, the connecting homomorphism $\delta : H^1(G,C) \to H^2(G,A)$ has the property that if $\delta(\xi) = 1$, then $\xi = 1$. But it need not be true that $\delta$ is an injective map of sets. So it is not clear, *a priori*, that $H^1\big(\mathrm{Gal}(\bar{K}/K), \mathrm{PGL}_2(\bar{K})\big)$ injects into $H^2\big(\mathrm{Gal}(\bar{K}/K), \bar{K}^*\big)$[2].

However, for these particular groups, it is true, and indeed more is true. Roughly speaking, the union of sets

$$\bigcup_{d \geq 1} H^1\big(\mathrm{Gal}(\bar{K}/K), \mathrm{PGL}_d(\bar{K})\big)$$

can be given the structure of a group, this group is isomorphic to the Brauer group of $K$, and under this identification we have a bijection

$$H^1\big(\mathrm{Gal}(\bar{K}/K), \mathrm{PGL}_d(\bar{K})\big) \xrightarrow{\sim} H^2\big(\mathrm{Gal}(\bar{K}/K), \bar{K}^*\big)[d] = \mathrm{Br}(K)[d].$$

For a discussion of this bijection, see [Serre, *Local Fields*, Chapter X, Section 5], and for a discussion of exact sequences of pointed sets coming from nonabelian cohomology, see [Serre, *Local Fields*, Appendix to Chapter VII], and in particular the two remarks at the end of the appendix.

**Page 206, Formula for $f^{-1}$ and $\phi^f$**
The inverse of the map

$$f(z) = \frac{\beta z + 1}{-\beta z + 1}$$

is

$$f^{-1}(z) = \frac{z - 1}{\beta z + \beta}.$$

So the formula for $\phi^f$ on line 7 is not correct, it should read

$$\phi^f(z) = \frac{\phi\left(\frac{\beta z+1}{-\beta z+1}\right) - 1}{\beta\phi\left(\frac{\beta z+1}{-\beta z+1}\right) + \beta}.$$

The formula for the twist $M_d^{(b)}$ of $z^d$ needs to be changed accordingly. More precisely, the formulas in the text are the reciprocals of the correct values. So this material should read:

For example, let $M_d(z) = z^d$ be the $d^{\text{th}}$-power map. Then a judicious use of the binomial theorem yields a formula for the $b$-twist $M_d^{(b)}$ of $M_d$,

$$M_d^{(b)}(z) = \sum_k \binom{d}{2k+1} b^k z^{2k+1} \Bigg/ \sum_k \binom{d}{2k} b^k z^{2k} \ .$$

In particular, the first few $b$-twists of $M_d$ are

$$M_2^{(b)}(z) = \frac{2z}{1+bz^2}, \qquad M_3^{(b)}(z) = \frac{3z+bz^3}{1+3bz^2}, \qquad M_4^{(b)}(z) = \frac{4z+4bz^3}{1+6bz^2+b^2z^4}.$$

**Page 207, Definition of $G_\phi$**
It should be noted that $G_\phi$ is an open, and hence closed, subgroup of $\mathrm{Gal}(\bar{K}/K)$. To see this, let $E/K$ be a finite extension such that $\phi(z) \in E(z)$ and let $H = \mathrm{Gal}(\bar{K}/E)$. Then $H$ is an open neighborhood of the identity in $\mathrm{Gal}(\bar{K}/K)$, and for any $\sigma \in G_\phi$ and any $\tau \in H$ we have

$$\sigma\tau(\phi) = \sigma(\phi) = \phi^{g_\sigma},$$

so $\sigma H \subset G_\phi$.

**Page 208, Line $-5$**
It is not true that $\sigma \in G_\phi$ or that $G_\phi = \{1, \sigma\}$, since $\sigma \in \mathrm{Gal}(\mathbb{Q}(i)/\mathbb{Q})$, while $G_\phi \subset \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$. This sentence should read:

This shows that any extension of $\sigma$ to $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ is in $G_\phi$, so $G_\phi = \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ and $K_\phi = \mathbb{Q}$.

**Page 215, first displayed equation**
The divisor $D'$ has degree $-2$, so it should be

$$D' = -(Q_1) - (Q_2).$$

**Page 215, Proof of Proposition 4.91**
The proof that (c) implies (a) is somewhat confusing. It's probably clearer to prove that (c) implies (b), and then that (b) implies (a).

In order to prove that (c) implies (b), we use the argument that is given in the text to create a divisor $E$ of degree 1 that is defined over $K$. Then Riemann-Roch says that there is a function $f$ defined over $K$ with $\mathrm{div}(f) + E \ge 0$. Since $\mathrm{div}(f) + E$ has degree 1, this means that $\mathrm{div}(f) + E = (R)$ for some point $R$. The divisor $\mathrm{div}(f) + E$ is defined over $K$, i.e., it is invariant under the action of $\mathrm{Gal}(\bar{K}/K)$, so $R$ is in $C(K)$.

In order to prove that (b) implies (a), we are given a point $R \in C(K)$. Then Riemann-Roch says that there is a function $\psi$ defined over $K$ with $\mathrm{div}(\psi)+(R) \ge 0$. But $\mathrm{div}(\psi)+(R)$ has degree 1, so we have $\mathrm{div}(\psi)+(R) = (S)$ for some point $S$. Thus $\mathrm{div}(\psi) = (S) - (R)$, so $\psi : C \to \mathbb{P}^1$ has a single pole,

so $\psi$ is a map of degree 1. Since $C$ is smooth, the map $\psi$ is an isomorphism, so $C$ is isomorphic over $K$ to $\mathbb{P}^1$.

**Page 218, Section 4.11**
The discussion of minimal models can be much simplified once one notes that if $R$ is a PID with fraction field $K$, then every element of $GL_2(K)$ can be written in the form

$$\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \quad \text{with} \quad \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in \mathrm{GL}_2(R).$$

Since conjugation by a matrix in $\mathrm{GL}_2(R)$ doesn't change the valuation of the minimal discriminant, it follows that any rational map $\phi \in K(z)$ can be put into minimal form by conjugating by an affine transformation $f(z) = \alpha z + \beta$. This makes minimal discriminants and minimal models much easier to compute. It also means that the distinction between "minimal" and "affine minimal" in the discusssion of integer points is moot, at least over a PID such as $\mathbb{Z}$. Bruin and Molnar [a] give an algorithm to compute minimal models, and there is related work by Rumely [b] that ties minimal models in with Berkovich space. Bruin and Molnar also give minimal maps with many integer points.
[a] N. Bruin and A. Molnar, Minimal models for rational functions in a dynamical setting, *LMS J. Comput. Math.* **15** (2012), 400–417.
[b] R. Rumely, The Minimal Resultant Locus, 2013, `arXiv:1304.1201`.

**Page 221, Conjecture 4.97**
Patrick Ingram notes that this conjecture is not true. For example, the rational map $\phi(z) = z^2 + p^{-n}$ with $p \geq 3$ has minimal resultant that grows with $n$, while it has bad reduction only at $p$. (For example, if $n$ is even, then conjugation by $z \to z/p^{n/2}$ leads to a rational map with resultant $p^n$, which presumably is the minimal resultant.)

**Page 225, Exercise 4.6(b)**
Remove the $(z)$. Thus the displayed equation should read

$$\Phi^*_{n,\phi^p} = \phi^*_{n,\phi} \Phi^*_{np,\phi}.$$

**Page 225, Exercise 4.7**
$\lambda_\phi(\alpha)$ should be $\lambda_\alpha(\phi)$.

**Page 227, Exercise 4.9(b)**
The exponent on the resultant is wrong. It should be $d^{n-1}(d^n - 1)/(d-1)$. So the formula should read

$$\mathrm{Res}(F_n, G_n) = \mathrm{Res}(F, G)^{d^{n-1}(d^n-1)/(d-1)}.$$

**Page 230, Exercise 4.20(a,b)**
The curve $X_0(4)$ has genus 0, so both $X_0(4)(\mathbb{Q})$ and $Y_0(4)(\mathbb{Q})$ are infinite.

**Page 236, Exercise 4.30**
Parts (f) and (g) refer to $\mathcal{M}_d^{\mathrm{BiCrit}}$ without having defined it as the quotient of $\mathrm{BiCrit}/\mathrm{PGL}_2$. So (b) should include an additional part to deduce that $\mathrm{BiCrit}/\mathrm{PGL}_2$ exists as a geometric quotient and denoting this quotient by $\mathcal{M}_d^{\mathrm{BiCrit}}$.

**Page 236, Exercise 4.42**
Remove the assumption that $\mathrm{Aut}(X)$ is abelian (see the correction to the definition of the cohomology set on page 202).

**Page 247, Remark 5.9**
Start this remark with the following assumption:

Let $r \in |K^*|$.

This is necessary, because if $r \notin |K^*|$, then one can have $\sum a_i(z-a)^i$ converging on $\bar{D}(a,r) = D(a,r)$ even though $|a_i|r^i \not\to 0$. Here is an example (shown to me by X. Faber) which might make a good exercise.

Let $K = \mathbb{C}_p$, $\alpha \in \mathbb{R} \smallsetminus \mathbb{Q}$, and $r = p^\alpha$. Let $\alpha_i \in \mathbb{Q}$ be a sequence monotonically increasing to $\alpha$. For each $i$, choose $a_i \in \mathbb{C}_p$ satisfying $|a_i| = p^{-i\alpha_i}$. Then $i(\alpha - \alpha_i) > 0$ for all $i$, so

$$|a_i|r^i = p^{i(\alpha-\alpha_i)} > 1.$$

On the other hand, if $z \in \bar{D}(a,r)$, then $|z-a| = p^\beta < r = p^\alpha$ for some $\beta \in \mathbb{Q}$. Since $\beta < \alpha$, we ahve

$$|a_i| \cdot |z-a|^i = p^{i(\beta-\alpha_i)} \sim p^{i(\beta-\alpha)} \to 0.$$

Thus $\sum a_i(z-z)^i$ converges on $\bar{D}(a,r)$.

**Page 302, Line** $-10$
$\bigwedge_{0,0}^\circ$ should be $\bigwedge_{0,1}^\circ$. So the sentence should read:

... attach one extra copy of $\bigwedge_{0,1}^\circ$ running vertically upward from the Gauss point $\xi_{0,1}$.

**Page 303, Remark 5.70**
The point $\xi_{p^{-2},p^{-3}}$ should probably be identified with the point $\xi_{p^2,p^{-7}}$, since in general the identification should be between $\xi_{a,r}$ and $\xi_{a^{-1},r/|a|^2}$.

**Page 308, Line** $-9$
$\bigwedge_{0,0}^\circ$ should be $\bigwedge_{0,1}^\circ$. So the sentence should read:

We have seen that every point in $\bigwedge_{0,1}^\circ$ is attracted to $\xi_{0,0}$,...

**Page 311, Theorem 5.82(d)**
This is ungrammatical. It should read:

(d) Either the Julia set $\mathcal{J}^{\mathcal{B}}(\phi)$ is connected, or else it has infinitely many connected components.

**Page 312, Exercise 5.5**
"outlines a prove" should be "outlines a proof"

**Page 313, Exercise 5.8**
The $n!$ should be in the numerator, which means that the estimate actually gets better as $n \to \infty$. In other words, it should read: Prove that

$$\left|\frac{d^n\phi}{dz^n}(a)\right| \le \frac{s|n!|}{r^n} \qquad \text{for all } n \ge 1.$$

(Note that as $n$ increases, the estimate becomes better, since $|n!| \to 0$ as $n \to \infty$.)

**Page 364, Second displayed equation**
The critical values $0, 1, \kappa, \infty$ are the critical values of $x$, not $\pi$, so the points $T_0, \ldots$ are the inverse images using $x$, not $\pi$. Thus this displayed equation should read as follows:

$$T_0 = x^{-1}(0), \quad T_1 = x^{-1}(1), \quad T_\kappa = x^{-1}(\kappa), \quad \mathcal{O} = x^{-1}(\infty).$$

**Page 364, Section 6.6**
The term *rigid Lattès map* is never defined! And many of the results in this section apply to all Lattès maps. This suggests that flexible Lattès maps are rigid. So probably Section 6.6 should be entitled "General Lattès Maps" and the term *rigid* should be reserved for Lattès maps associated to $P \mapsto [\alpha]P+T$ where $\alpha \notin \mathbb{Z}$. The reason that these are "rigid' is because they do not vary in a continuous family, whereas Lattès maps associated to $P \mapsto [m]P + T$ can be varied continuously by taking a familly of elliptic curves.

**Page 365, Theorem 6.57**
Specify that the map $E \to E/\Gamma$ is the natural projection $P \mapsto P \bmod \Gamma$.

**Page 381, Exercise 6.12**
The curve $E$ should be $y^2 = x^3 + b$, not $y^2 = x^3 + 1$.

**Page 383, Exercise 6.22(c)**
The sentence starting "More precisely" is actually the contrapositive of Proposition 6.55, not the converse. Replace that sentence with the following:

More precisely, if $\phi$ is a Lattès map fitting into a reduced Lattès diagram (6.37) and if $\phi^f$ has good reduction for some $f \in \mathrm{PGL}_2(K)$, does the elliptic curve $E$ also have good reduction?

**Page 384, Exercise 6.24(b)**
It should be "nontrivial element of $\Gamma$", not "nontrivial element of $\xi$".

**Page 384, Exercise 6.24(b)**
The quantity $e_P(\pi)$ should be the number of elements in the set, not the set

itself. Thus it should read:

$$e_P(\pi) = \#\big\{\xi \in \Gamma : [\xi]P = P\big\}.$$

**Page 384, Exercise 6.24(c)**
Change "$T = 0$" to "$T = \mathcal{O}$".

**Page 393, Example 7.9, Line 2**
Change "Dehomogenizing" by "Homogenizing".

**Page 394, Theorem 7.10**
Parts of this theorem are false or not well-defined. If $\phi$ has degree 1, then $Z(\phi) = Z(\phi^{-1}) = \emptyset$, so $\ell_1$ and $\ell_2$ are not defined. And if $\phi$ is the identity map, then (c) is clearly false. So change the statement of the theorem to read:

Let $\phi : \mathbb{A}^N \to \mathbb{A}^N$ be a regular affine automorphism of degree at least 2.

**Page 395, Line** $-15$
Change "$\deg(\phi^n) = \deg(\phi)$" to "$\deg(\phi^n) = \deg(\phi)^n$". (Missing superscript $n$)

**Page 404, Definition in middle of page**
The prime divisors $W_1, \ldots, W_n$ are not *disjoint* as subsets of $V$, they are really the *distinct* irreducible subvarieties of $\phi^{-1}(W')$. So change "disjoint union of prime divisors" to "union of distinct prime divisors".

**Page 406, Line** $-10$
$\phi^* H$ should be $i^* H$, so the sentence should read "... then $i^* H$ is a very ample divisor on $V$."

**Page 434, Exercise 7.28**
In (a), it should read "Deduce that $\psi'(E)$ is a curve $C$ on $S'$."

In (b), the letters "$bfB$" should be the symbol $\mathbf{B}$. This occurs four times.

Also in (b), it should say $C \cdot K_{S'} \le E \cdot K_T = -1$. (It currently says $E \cdot K_S$, which makes no sense, since $E$ is a divisor on $T$, not on $S$.)

**Page 438, Exercise 7.41b**
Specify that the Zariski closed subset is a proper subset, i.e., of all of $\mathbb{P}^{26}$.

**Page 438, Exercise 7.42c**
"such at" should be "such as".

**Page 459, References**
The article "Equidistribution and integral points for Drinfeld modules" that is attributed to H. Glockner (ArXiv:math.NT.0609120) should be attributed to D. Ghioca and T. Tucker.

**Page 484, Index**
The index entry for "exercise, hard" has a TEX error (caused by an extra backslash before textbf)

**Page 487, Index**
The index entry for "hard problem" has a TEX error (caused by an extra backslash before textbf)