

Errata and Corrections to
The Arithmetic of Dynamical Systems
1st Edition

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Acknowledgements Page vi

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Page 4, line 25

The sentence “In particular, we prove that \mathcal{M}_2 is an isomorphism to the affine plane \mathbb{A}^2 .” should read “In particular, we prove that \mathcal{M}_2 is isomorphic to the affine plane \mathbb{A}^2 .”

Page 4, line –6

The book says that $\mathbb{P}^1(\mathbb{C}_p)$ is not Hausdorff, but it is clear that $\mathbb{P}^1(\mathbb{C}_p)$ is a Hausdorff space.

Page 9, Line –1 and Page 63, Line 8

There is inconsistent notation in labeling the coefficients of rational functions. Page 9 gives a rational function as

$$\phi(z) = \frac{F(z)}{G(z)} = \frac{a_0 + a_1z + \cdots + a_dz^d}{b_0 + b_1z + \cdots + b_dz^d}$$

and page 63 gives a rational function as

$$\phi(z) = \frac{a_0z^d + a_1z^{d-1} + \cdots + a_{d-1}z + a_d}{b_0z^d + b_1z^{d-1} + \cdots + b_{d-1}z + b_d}$$

This should be made consistent throughout the book.

Page 48, line –1

$\mathbb{P}^1(k)$ should be $\mathbb{P}^N(k)$

Page 49, line 2

$\mathbb{P}^1(k)$ should be $\mathbb{P}^N(k)$

Page 49, line 1 of example 2.6

$\mathbb{P}^1(\mathbb{Q})$ should be $\mathbb{P}^3(\mathbb{Q})$

Page 51, Proof of Proposition 2.11

The inequalities in the first three lines of the proof should be reversed. Thus it should read

Write $P_i = [X_i, Y_i]$ with normalized coordinates. If $v(X_1) < v(Y_1)$, we begin by applying the map $f = Y/X \in \text{PGL}_2(R)$ to each of the three points, so we may assume that $v(X_1) \geq v(Y_1)$. Since the coordinates are normalized, this implies that $v(Y_1) = 0$, so Y_1 is a unit.

Page 62, line 1

The penultimate ϕ on this line should have a tilde. Thus it should read

$$\tilde{P} = \tilde{\phi}^n(\tilde{P}) = \tilde{\phi}^r \circ \underbrace{\tilde{\phi}^m \circ \dots \circ \tilde{\phi}^m}_{k \text{ iterations}}(\tilde{P}) = \tilde{\phi}^r(\tilde{P}).$$

Page 79, Exercise 2.24

Add the assumption that the residue characteristic is not equal to 2.

Page 84, line -9

There is a missing absolute value symbol in the middle part of the displayed formula. It should read

$$\prod_{v \in M_K} \max_i \{|\alpha x_i|_v\}^{n_v} = \prod_{v \in M_K} |\alpha|_v^{n_v} \max_i \{|x_i|_v\}^{n_v} = \prod_{v \in M_K} \max_i \{|x_i|_v\}^{n_v}.$$

Page 85, Second displayed formula

$H(P)$ should be $H_{\mathbb{Q}}(P)$, since $H(P)$ hasn't been defined yet. So this line should read

$$\{P \in \mathbb{P}^N(\mathbb{Q}) : H_{\mathbb{Q}}(P) \leq B\}$$

Page 86, Third line of first displayed formula

There's a missing n_v in the exponent. So this full display should read

$$\begin{aligned} H_L(\sigma(P)) &= \prod_{v \in M_L} \max\{|\sigma(x_0)|_v, \dots, |\sigma(x_N)|_v\}^{n_v} \\ &= \prod_{v \in M_L} \max\{|x_0|_{\sigma(v)}, \dots, |x_N|_{\sigma(v)}\}^{n_{\sigma(v)}} \\ &= \prod_{v \in M_L} \max\{|x_0|_v, \dots, |x_N|_v\}^{n_v} \\ &= H_L(P). \end{aligned}$$

Page 92, bottom

The coefficients of the polynomials g_{ij} whose existence comes from the Nullstellensatz lie in \bar{K} , but the subsequent proof suggests that the g_{ij} have coefficients in K . There are two ways around this apparent problem. The easiest is to simply replace K with the finite extension field obtained by adjoining all of the coefficients of all of the g_{ij} 's. The compatibility of the height under field extensions says that we can do this. Alternatively, it is not hard (using some elementary algebraic number theory) to show that in fact one can choose the g_{ij} to have coefficients in K .

Page 102, First displayed formula

This is supposed to be the absolute height, so there should be a factor of $1/[K : \mathbb{Q}]$. Thus this line should read

$$h(P) = h(\alpha) = \frac{1}{[K : \mathbb{Q}]} \sum_{v \in M_K} n_v \log \max\{|\alpha|_v, 1\},$$

Page 103, Theorem 3.29, Equation (3.17)

This is supposed to be the absolute height, so there should be a factor of $1/[K : \mathbb{Q}]$. Thus this line should read

$$\hat{h}_\phi(\alpha) = \frac{1}{[K : \mathbb{Q}]} \sum_{v \in M_K} n_v \hat{\lambda}_{\phi,v}(\alpha) \quad \text{for all } \alpha \in \mathbb{P}^1(K) \setminus \{\infty\}.$$

Page 105, Theorem 3.35

The case $B = 0$ must be excluded.

Page 105, Proof of Theorem 3.35

Need to explain why the plane curves $G_1(X, Y) = B_1$ and $G_2(X, Y) = B_2$ have no common components.

Page 108, Theorem 3.40 (Roth's theorem)

Need to specify that $\alpha_v \notin K$, so the statement of the theorem should read "choose an algebraic number $\alpha_v \in \bar{K} \setminus K$."

Page 111, 2nd displayed equation

The variable should be z , not x . Thus

$$\phi(z) = \frac{899z^2 - 2002z + 275}{33z^2 - 584z + 275}.$$

Page 111, Proposition 3.46

The rational map is supposed to have degree d . So it should read:

Proposition 3.46. *For all integers $N \geq 0$ and $d \geq 2$ there exists a rational map $\phi(z) \in \mathbb{Q}(z)$ of degree d with the following properties:*

Page 113, Table 3.1, Caption

The caption should be $\mathcal{O}_\phi(1)$, not $\mathcal{O}_1(\phi)$.

Page 114, Line -8

It is not true that

$$\frac{1}{|a_n|^\epsilon} \geq \left| \frac{b_n}{a_n} \right|$$

is a consequence of $|a_n| \geq |b_n|^{1+\epsilon}$. So this needs to be adjusted. What is true is that

$$|a_n| \geq |b_n|^{1+\epsilon} \implies \frac{1}{|a_n|^{\epsilon/(1+\epsilon)}} \geq \left| \frac{b_n}{a_n} \right|.$$

We may assume that $\epsilon \leq \frac{1}{2}$, which implies that

$$\frac{\epsilon}{1+\epsilon} \geq \frac{2}{3}\epsilon.$$

So assuming $\epsilon \leq \frac{1}{2}$, we have

$$|a_n| \geq |b_n|^{1+\epsilon} \implies \frac{1}{|a_n|^{2\epsilon/3}} \geq \left| \frac{b_n}{a_n} \right|.$$

So can change the exponent on line -8 on page 114 to $\frac{2}{3}\epsilon$, and then line 3 on page 115 needs to be changed to

$$\log |a_n| \leq \frac{6}{\epsilon} \log(C_5^{-1}).$$

Page 115, Line 1

It should be \geq instead of $=$.

Page 115, Line 3

It should be C_5^{-1} instead of C_5 , so the inequality should read

$$\log |a_n| \leq \frac{2}{\epsilon} \log(C_5^{-1}).$$

(However, see also the correction listed above for page 114, line -8.)

Page 118, Equation (3.33)

The “ f ” should be “ ϕ ”. Thus it should read

$$e_P(\phi^2) = e_P(\phi)e_{\phi(P)}(\phi) = e_0e_1 \leq d^2 - d.$$

Page 120, Line 3 (Second displayed equation)

In the definition of $N(\phi, \alpha, \epsilon, \beta)$, the β_n should just be β . Thus this line should read

$$N(\phi, \alpha, \epsilon, \beta) = \left\{ n \in N(\phi, \alpha, \epsilon) : \rho(\phi^{n-m}(\alpha), \beta)^{e_\infty(\phi^m)} \leq C_1 \rho(\phi^n(\alpha), \infty) \right\},$$

Page 121, First displayed equation

The constants C_7, C_8, C_9 should be C_8, C_9, C_{10} , since C_7 was already used on the previous page. Thus the displayed equation should read

$$\begin{aligned} \frac{1}{|a_n|^\epsilon} &\geq \rho(\phi^n(\alpha), \infty) && \text{from (3.34),} \\ &\geq C_8 \rho(\phi^{n-m}(\alpha), \beta)^{e_\infty(\phi^m)} && \text{from (3.36),} \\ &= C_8 \left(\frac{b_{n-m}}{\sqrt{a_{n-m}^2 + b_{n-m}^2}} \right)^{e_\infty(\phi^m)} && \text{definition of } \rho, \\ &\geq \frac{C_9}{H(\phi^{n-m}(\alpha))^{e_\infty(\phi^m)}} && \text{definition of height, where note} \\ &&& \text{that } b_{n-m} \neq 0, \text{ since } \alpha \text{ is wander-} \\ &&& \text{ing and } \infty \text{ is periodic,} \\ &\geq \frac{C_{10}}{H(\phi^n(\alpha))^{e_\infty(\phi^m)/d^m}} && \text{from Theorem 3.11,} \\ &\geq \frac{C_{10}}{H(\phi^n(\alpha))^{\epsilon/6}} && \text{from (3.35),} \\ &= \frac{C_{10}}{|a_n|^{\epsilon/6}} && \text{since } H(\phi^n(\alpha)) = H(a_n/b_n) = |a_n|. \end{aligned}$$

Page 128, footnote

Add the assumption that the boundary of U has measure 0. So the footnote should read: “Recall that a sequence of measures μ_i on a compact space X converges weakly to μ if for every Borel-measurable set U with boundary satisfying $\mu(\partial U) = 0$, the sequence of values $\mu_i(U)$ converges to $\mu(U)$ as $i \rightarrow \infty$.”

Page 130, Line –9 before Proposition 3.63

The point P has not been defined. So replace the sentence starting “Choose a prime ideal \mathfrak{P} . . .” with the following:

Let $P \in \text{Per}_n^{**}(\phi)$ be a point of exact period n , and choose a prime ideal \mathfrak{P} in $K_{\phi,n}^0(P)$ lying above \mathfrak{p} .

Page 131, Corollary 3.64

It is not clear why Proposition 3.63 implies that the points of formal period n remain distinct when reduced modulo \mathfrak{p} for the indicated primes. It is true that Proposition 3.63 says (more-or-less) that the point and its reduction have the same period, but why does that imply injectivity?

Page 135, Exercise 3.3

The quantity $D(P)$ is not defined. It is supposed to be the degree of the field of definition of P . So add the following at the beginning of this exercise:

For any $P \in \mathbb{P}^N(\bar{\mathbb{Q}})$, let $D(P) = [\mathbb{Q}(P) : \mathbb{Q}]$ be the degree of the field of definition of P .

Page 137, Exercise 3.9

Part (d) should certainly be marked as a hard problem, and possibly (c) should, too.

Page 137, Exercise 3.13

The stated result is true for any perfect field. For the first part of the proof, one assumes that $\phi^n(\sigma(P)) = \phi^m(P)$ for some $n \neq m$ and some $\sigma \in \text{Gal}(\bar{K}/K)$ and shows that P is preperiodic. Over number fields, I had in mind the following argument using the Galois invariance of the canonical height:

$$\begin{aligned}\hat{h}(\phi^n(\sigma(P))) &= \hat{h}(\phi^m(P)), \\ d^n \hat{h}(\sigma(P)) &= d^m \hat{h}(P), \\ d^n \hat{h}(P) &= d^m \hat{h}(P), \\ \hat{h}(P) &= 0,\end{aligned}$$

and hence P is preperiodic. However, one can instead use a simple combinatorial argument and the fact that $\{\sigma^i P : i \geq 0\}$ is finite to prove this fact in general. Then the rest of the proof works over any perfect field.

Page 138, Exercise 3.17

Specify that the map ϕ has degree $d \geq 2$.

Page 139, Exercise 3.21

(b) The displayed equation

$$e^{n^2 \hat{h}_\phi(\alpha)} - \phi^n(2)$$

should be

$$e^{2^n \hat{h}_\phi(\alpha)} - \phi^n(2).$$

(In other words, the exponent should have 2^n , not n^2 .)

(c) “is the closest integer to H^{n^2} ” should be “is the closest integer to H^{2^n} ”

page 139, Exercise 3.22

H^{n^2} should be H^{d^n} .

page 139, Exercise 3.23

In the displayed equation, the exponent should have d^n instead of n^2 . Thus it should read

$$\left| e^{d^n \hat{h}_\phi(\alpha)} + \frac{a}{d} - \phi^n(\alpha) \right| \leq \epsilon \quad \text{for all } n \geq 0.$$

(Also move the d^n to the other side of the $\hat{h}_\phi(\alpha)$, which is not an error, but makes the notation in Exercise 3.23 consistent with the notation in Exercise 3.21(b).)

Page 140, Exercise 3.24(b)

This is supposed to be the absolute height, so there should be a factor of $1/[K : \mathbb{Q}]$. Thus this line should read

$$\hat{h}_\phi(\alpha) = \frac{1}{[K : \mathbb{Q}]} \sum_{v \in M_K} n_v \hat{\lambda}_{\phi, v}(\alpha).$$

Page 140, Exercise 3.32(a)

The bound should be $\sqrt{4|B|/3}$ (missing absolute value sign).

Page 144, Line 13

Instead of $\alpha \in \mathbb{P}^N(\mathbb{Q}) \setminus \text{PrePer}(\phi)$, it should say $\alpha \in \mathbb{P}^1(\mathbb{Q}) \setminus \text{PrePer}(\phi)$.

Page 146, Exercise 3.49(a)

It should be “for some $e \in \mathbb{Q}$,” not “for some $e \in \mathbb{Q}(c)$.”

Page 150, Definition

Reverse the order of the 2nd and 3rd bullet items to match the order of Per_n , Per_n^* , and Per_n^{**} .

Page 151, Theorem 4.5(b) and proof

Should not use $\lambda(P)$ to denote the multiplier at P , since that’s not consistent with earlier notation. Use either λ_P and $\lambda_P(\phi)$.

Page 153, Line 2

Insert “be” between “may” and “done”.

Page 158, Line –8

Replace “primitive- n periodic” with “primitive n -periodic”. (Misplaced hyphen)

Page 159, Line 12 (third displayed equation)

$$\psi_t(z) = z^2 - (t + t^{-1} - 1)z \quad \text{and} \quad \lambda(t) = t - 1$$

The function should be $z^2 - \dots$, not $z^2 + \dots$. So the full line should read

Page 159

Should not use λ to denote a map, since too easily confused with the use of λ as the multiplier of a map.

Page 160, equation (4.17)

The righthand side should be $\Phi_{n,P}^*$, i.e., add a star.

Page 161, first paragraph

It is not clear, *a priori*, that ϕ induces an automorphism of exact order n on $Y_0(n)$. One needs to know that there is at least one value of c for which $\phi_c(z) = z^2 + c$ has a point of primitive period n . The description of the bifurcation polynomials in Section 4.2.4 and the identification of their roots with a finite set of points of the Mandelbrot set gives the desired result.

Page 174, Bottom of page

The action of PGL_2 on $\mathbb{Q}[\mathrm{Rat}_d]$ is defined as

$$R^f(\phi) = R(\phi^f).$$

However, this is a *left action*, i.e., $R^{fg} = (R^g)^f$, so the notation is confusing. It might be better to write the action as fR . Otherwise add a note indicating that it is a left action.

Page 180, line 3

It should be $\lambda_P(\phi) \in \mathbb{C}$, not $\lambda_P(\phi) \in \mathbb{C}^*$, i.e., the multiplier may be 0.

Page 180, line -3

It is true that $\lambda_{P_i}(\phi)$ is integral over $\mathbb{Q}[\mathrm{Rat}_d]$, but the ring listed here is not $\mathbb{Q}[\mathrm{Rat}_d]$. The correct description of $\mathbb{Q}[\mathrm{Rat}_d]$ in Proposition 4.27 on page 169. Thus

$$\mathbb{Q}[\mathrm{Rat}_d] = \mathbb{Q}[a_0, \dots, b_d, \mathrm{Res}(F_{\mathbf{a}}, F_{\mathbf{b}})^{-1}]^{(0)},$$

where the superscript (0) denotes the elements of degree 0, i.e., quotients of homogeneous polynomials of the same degree.

Page 187, lines 5 and 6

Change M_4 to \mathcal{M}_4

Page 187, lines 18 and 20

Change σ_{d,N^*} to $\sigma_{d,N}^*$

Page 191, Proof of (a)

Even assuming $\lambda_1\lambda_2 \neq 1$ and using (4.37), it does not follow that $\lambda_1 \neq 1$ and $\lambda_2 \neq 1$. However, all that we really need is that the fixed points associated to λ_1 and λ_2 are distinct. So replace the first two sentences of the proof of (a) with the following:

(a) For this part we assume that $\lambda_1\lambda_2 \neq 1$. If $\lambda_1 \neq \lambda_2$, then the fixed points associated to λ_1 and λ_2 are clearly distinct. But if $\lambda_1 = \lambda_2$, then our assumption implies that λ_1 and λ_2 are not equal to 1, so again the fixed points must be distinct. Hence we can find an element of $\mathrm{PGL}_2(\mathbb{C})$ that moves them to 0 and ∞ , respectively.

Page 192, Line –10

The words “2-equivalent” should read “ PGL_2 -equivalent.” So the entire phrase reads:

Then Lemma 4.59(a) says that ϕ_1 and ϕ_2 are PGL_2 -equivalent to the function $(z^2 + \lambda_1 z)/(\lambda_2 z + 1), \dots$

Page 199, Line 15

”and those that do, fall into finitely many...” Remove the comma after “do”.

Page 202, Definition

The definition of cohomologous 1-cycles is not correct if A is not abelian. The correct definition is that the 1-cocycles g_1 and g_2 are cohomologous if there is an $f \in A$ such that

$$g_{2,\sigma} = f g_{1,\sigma} \sigma(f^{-1}) \quad \text{for all } \sigma \in G.$$

Note that if A is abelian, then this is equivalent to $g_{1,\sigma}^{-1} g_{2,\sigma} = f \sigma(f^{-1})$ being a coboundary.

Page 202, Remark 4.78

Using the correct definition of cohomologous cycles, the indicated map is a well-defined injection of $\mathrm{Twist}(X/K)$ into $H^1(\mathrm{Gal}(\bar{K}/K), \mathrm{Aut}(X))$ even in the case that $\mathrm{Aut}(X)$ is nonabelian.

Page 203, Theorem 4.79

“Let $\phi(z) \in K(z)$ be a nonzero rational map...” should be “Let $\phi(z) \in K(z)$ be a nonconstant rational map...”

Page 204, Remark 4.80

It is true that $H^1(\mathrm{Gal}(\bar{K}/K), \mathrm{PGL}_2(\bar{K}))$ injects into $H^2(\mathrm{Gal}(\bar{K}/K), \bar{K}^*)[2]$, but it is not a formal argument using exact sequences. In general, if B is a non-abelian group on which G acts, then the cohomology group $H^1(G, B)$ is a pointed set. Then for any exact sequence of groups on which G acts $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$, assuming A is in the center of B , we get an exact sequence of pointed sets

$$H^1(G, A) \rightarrow H^1(G, B) \rightarrow H^1(G, C) \rightarrow H^2(G, A).$$

However, an exact sequence of pointed sets only means that kernels equal images, and a kernel is the inverse image of the distinguished point. So for example, the connecting homomorphism $\delta : H^1(G, C) \rightarrow H^2(G, A)$ has the property that if $\delta(\xi) = 1$, then $\xi = 1$. But it need not be true that δ is an injective map of sets. So it is not clear, *a priori*, that $H^1(\mathrm{Gal}(\bar{K}/K), \mathrm{PGL}_2(\bar{K}))$ injects into $H^2(\mathrm{Gal}(\bar{K}/K), \bar{K}^*)[2]$.

However, for these particular groups, it is true, and indeed more is true. Roughly speaking, the union of sets

$$\bigcup_{d \geq 1} H^1(\mathrm{Gal}(\bar{K}/K), \mathrm{PGL}_d(\bar{K}))$$

can be given the structure of a group, this group is isomorphic to the Brauer group of K , and under this identification we have a bijection

$$H^1(\mathrm{Gal}(\bar{K}/K), \mathrm{PGL}_d(\bar{K})) \xrightarrow{\sim} H^2(\mathrm{Gal}(\bar{K}/K), \bar{K}^*)[d] = \mathrm{Br}(K)[d].$$

For a discussion of this bijection, see [Serre, *Local Fields*, Chapter X, Section 5], and for a discussion of exact sequences of pointed sets coming from nonabelian cohomology, see [Serre, *Local Fields*, Appendix to Chapter VII], and in particular the two remarks at the end of the appendix.

Page 207, Definition of G_ϕ

It should be noted that G_ϕ is an open, and hence closed, subgroup of $\mathrm{Gal}(\bar{K}/K)$. To see this, let E/K be a finite extension such that $\phi(z) \in E(z)$ and let $H = \mathrm{Gal}(\bar{K}/E)$. Then H is an open neighborhood of the identity in $\mathrm{Gal}(\bar{K}/K)$, and for any $\sigma \in G_\phi$ and any $\tau \in H$ we have

$$\sigma\tau(\phi) = \sigma(\phi) = \phi^{g_\sigma},$$

so $\sigma H \subset G_\phi$.

Page 208, Line -5

It is not true that $\sigma \in G_\phi$ or that $G_\phi = \{1, \sigma\}$, since $\sigma \in \mathrm{Gal}(\mathbb{Q}(i)/\mathbb{Q})$, while $G_\phi \subset \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$. This sentence should read:

This shows that any extension of σ to $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ is in G_ϕ , so $G_\phi = \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ and $K_\phi = \mathbb{Q}$.

Page 215, first displayed equation

The divisor D' has degree -2 , so it should be

$$D' = -(Q_1) - (Q_2).$$

Page 221, Conjecture 4.97

Patrick Ingram notes that this conjecture is not true. For example, the rational map $\phi(z) = z^2 + p^{-n}$ with $p \geq 3$ has minimal resultant that grows with n , while it has bad reduction only at p . (For example, if n is even, then conjugation by $z \rightarrow z/p^{n/2}$ leads to a rational map with resultant p^n , which presumably is the minimal resultant.)

Page 225, Exercise 4.6(b)

Remove the (z) . Thus the displayed equation should read

$$\Phi_{n,\phi^p}^* = \phi_{n,\phi}^* \Phi_{np,\phi}^*.$$

Page 225, Exercise 4.7

$\lambda_\phi(\alpha)$ should be $\lambda_\alpha(\phi)$.

Page 227, Exercise 4.9(b)

The exponent on the resultant is wrong. It should be $d^{n-1}(d^n - 1)/(d - 1)$. So the formula should read

$$\text{Res}(F_n, G_n) = \text{Res}(F, G)^{d^{n-1}(d^n - 1)/(d - 1)}.$$

Page 236, Exercise 4.30

Parts (f) and (g) refer to $\mathcal{M}_d^{\text{BiCrit}}$ without having defined it as the quotient of $\text{BiCrit}/\text{PGL}_2$. So (b) should include an additional part to deduce that $\text{BiCrit}/\text{PGL}_2$ exists as a geometric quotient and denoting this quotient by $\mathcal{M}_d^{\text{BiCrit}}$.

Page 236, Exercise 4.42

Remove the assumption that $\text{Aut}(X)$ is abelian (see the correction to the definition of the cohomology set on page 202).

Page 247, Remark 5.9

Start this remark with the following assumption:

Let $r \in |K^*|$.

This is necessary, because if $r \notin |K^*|$, then one can have $\sum a_i(z - a)^i$ converging on $\bar{D}(a, r) = D(a, r)$ even though $|a_i|r^i \not\rightarrow 0$. Here is an example (shown to me by X. Faber) which might make a good exercise.

Let $K = \mathbb{C}_p$, $\alpha \in \mathbb{R} \setminus \mathbb{Q}$, and $r = p^\alpha$. Let $\alpha_i \in \mathbb{Q}$ be a sequence monotonically increasing to α . For each i , choose $a_i \in \mathbb{C}_p$ satisfying $|a_i| = p^{-i\alpha_i}$. Then $i(\alpha - \alpha_i) > 0$ for all i , so

$$|a_i|r^i = p^{i(\alpha - \alpha_i)} > 1.$$

On the other hand, if $z \in \bar{D}(a, r)$, then $|z - a| = p^\beta < r = p^\alpha$ for some $\beta \in \mathbb{Q}$. Since $\beta < \alpha$, we have

$$|a_i| \cdot |z - a|^i = p^{i(\beta - \alpha_i)} \sim p^{i(\beta - \alpha)} \rightarrow 0.$$

Thus $\sum a_i(z - a)^i$ converges on $\bar{D}(a, r)$.

Page 302, Line -10

$\mathbb{A}_{0,0}^\circ$ should be $\mathbb{A}_{0,1}^\circ$. So the sentence should read:

... attach one extra copy of $\mathbb{A}_{0,1}^\circ$ running vertically upward from the Gauss point $\xi_{0,1}$.

Page 303, Remark 5.70

The point $\xi_{p^{-2}, p^{-3}}$ should probably be identified with the point $\xi_{p^2, p^{-7}}$, since in general the identification should be between $\xi_{a,r}$ and $\xi_{a^{-1}, r/|a|^2}$.

Page 308, Line -9

$\mathbb{A}_{0,0}^\circ$ should be $\mathbb{A}_{0,1}^\circ$. So the sentence should read:

We have seen that every point in $\mathbb{A}_{0,1}^\circ$ is attracted to $\xi_{0,0}, \dots$

Page 311, Theorem 5.82(d)

This is ungrammatical. It should read:

(d) Either the Julia set $\mathcal{J}^B(\phi)$ is connected, or else it has infinitely many connected components.

Page 364, Second displayed equation

The critical values $0, 1, \kappa, \infty$ are the critical values of x , not π , so the points T_0, \dots are the inverse images using x , not π . Thus this displayed equation should read as follows:

$$T_0 = x^{-1}(0), \quad T_1 = x^{-1}(1), \quad T_\kappa = x^{-1}(\kappa), \quad \mathcal{O} = x^{-1}(\infty).$$

Page 364, Section 6.6

The term *rigid Lattès map* is never defined! And many of the results in this section apply to all Lattès maps. This suggests that flexible Lattès maps are rigid. So probably Section 6.6 should be entitled “General Lattès Maps” and the term *rigid* should be reserved for Lattès maps associated to $P \mapsto [\alpha]P + T$ where $\alpha \notin \mathbb{Z}$. The reason that these are “rigid” is because they do not vary in a continuous family, whereas Lattès maps associated to $P \mapsto [m]P + T$ can be varied continuously by taking a family of elliptic curves.

Page 365, Theorem 6.57

Specify that the map $E \rightarrow E/\Gamma$ is the natural projection $P \mapsto P \bmod \Gamma$.

Page 381, Exercise 6.12

The curve E should be $y^2 = x^3 + b$, not $y^2 = x^3 + 1$.

Page 383, Exercise 6.22(c)

The sentence starting “More precisely” is actually the contrapositive of Proposition 6.55, not the converse. Replace that sentence with the following:

More precisely, if ϕ is a Lattès map fitting into a reduced Lattès diagram (6.37) and if ϕ^f has good reduction for some $f \in \mathrm{PGL}_2(K)$, does the elliptic curve E also have good reduction?

Page 384, Exercise 6.24(b)

It should be “nontrivial element of Γ ”, not “nontrivial element of ξ ”.

Page 384, Exercise 6.24(b)

The quantity $e_P(\pi)$ should be the number of elements in the set, not the set itself. Thus it should read:

$$e_P(\pi) = \#\{\xi \in \Gamma : [\xi]P = P\}.$$

Page 384, Exercise 6.24(c)

Change “ $T = 0$ ” to “ $T = \mathcal{O}$ ”.

Page 393, Example 7.9, Line 2

Change “Dehomogenizing” by “Homogenizing”.

Page 394, Theorem 7.10

Parts of this theorem are false or not well-defined. If ϕ has degree 1, then $Z(\phi) = Z(\phi^{-1}) = \emptyset$, so ℓ_1 and ℓ_2 are not defined. And if ϕ is the identity map, then (c) is clearly false. So change the statement of the theorem to read:

Let $\phi : \mathbb{A}^N \rightarrow \mathbb{A}^N$ be a regular affine automorphism of degree at least 2.

Page 395, Line -15

Change “ $\deg(\phi^n) = \deg(\phi)$ ” to “ $\deg(\phi^n) = \deg(\phi)^n$ ”. (Missing superscript n)

Page 404, Definition in middle of page

The prime divisors W_1, \dots, W_n are not *disjoint* as subsets of V , they are really the *distinct* irreducible subvarieties of $\phi^{-1}(W')$. So change “disjoint union of prime divisors” to “union of distinct prime divisors”.

Page 406, Line -10

ϕ^*H should be i^*H , so the sentence should read “. . . then i^*H is a very ample divisor on V .”

Page 434, Exercise 7.28

In (a), it should read “Deduce that $\psi'(E)$ is a curve C on S' .”

In (b), the letters “ bfB ” should be the symbol **B**. This occurs four times.

Also in (b), it should say $C \cdot K_{S'} \leq E \cdot K_T = -1$. (It currently says $E \cdot K_S$, which makes no sense, since E is a divisor on T , not on S .)

Page 438, Exercise 7.41b

Specify that the Zariski closed subset is a proper subset, i.e., of all of \mathbb{P}^{26} .

Page 438, Exercise 7.42c

“such at” should be “such as”.

Page 459, References

The article “Equidistribution and integral points for Drinfeld modules” that is attributed to H. Glockner (ArXiv:math.NT.0609120) should be attributed to D. Ghioca and T. Tucker.

Page 484, Index

The index entry for “exercise, hard” has a \TeX error (caused by an extra backslash before `textbf`)

Page 487, Index

The index entry for “hard problem” has a \TeX error (caused by an extra backslash before `textbf`)