Solutions to Selected Homework Problems
Math 9 — Fall 2005

§4.3 How Derivatives Affect the Shape of a Graph

§4.3, #1. (a) Increasing $0 < x < 6$ and $8 < x < 9$.
(b) Decreasing $6 < x < 8$.
(c) Concave up on $2 < x < 4$ and $7 < x < 9$.
(d) Concave down on $0 < x < 2$ and $4 < x < 7$.
(e) Inflection points at $(2, 3)$ and $(4, \frac{9}{2})$ and $(7, 4)$

§4.3, #4. (a) If $f'(x) < 0$ for $x < c$ and $f'(x) > 0$ for $x > c$ (at nearby values of x), then f has a local local minimum at $x = c$.

If $f'(x) > 0$ for $x < c$ and $f'(x) < 0$ for $x > c$ (at nearby values of x), then f has a local local maximum at $x = c$.

(b) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local local minimum at $x = c$.

If $f'(c) = 0$ and $f''(c) < 0$, then f has a local local maximum at $x = c$.

If $f'(c) = 0$ and $f''(c) = 0$, then the second derivative test is inconclusive. Use the first derivative test, or check nearby values directly.

§4.3, #6. (a) $f(x)$ is increasing when $f'(x) > 0$. So $f(x)$ is increasing for $0 < x < 1$ and $3 < x < 5$. $f(x)$ is decreasing when $f'(x) < 0$. So $f(x)$ is decreasing for $1 < x < 3$ and $5 < x < 6$.

(b) f has a local max at $x = 1$ and at $x = 5$ by the first derivative test. f has a local min at $x = 3$ by the first derivative test.

§4.3, #12.

$$f(x) = 5 - 3x^2 + x^3 \quad f'(x) = -6x + 3x^2 = 3x(x-2) \quad f''(x) = -6 + 6x$$

(a) $f(x)$ is increasing for $x < 0$ and for $x > 2$. $f(x)$ is decreasing for $0 < x < 2$.

(b) There is a local maximum at $x = 0$ by the first derivative test. (Or by the second derivative test, since $f''(0) = -6 < 0$.) There is a local minimum at $x = 2$ by the first derivative test. (Or by the second derivative test, since $f''(2) = 6 > 0$.)

(c) The graph is concave up for $x > 1$ and it is concave down for $x < 1$. The point $(1, 3)$ is an inflection point.

§4.3, #17.

$$f(x) = xe^x \qquad f'(x) = (x+1)e^x \qquad f''(x) = (x+2)e^x$$

(a) Decreasing for $x < -1$. Increasing for $x > -1$.

(b) Local minimum at $x = -1$ by first derivative test (or by second derivative test, since $f''(-1) = e^{-1} > 0$).

(c) Concave down for $x < -2$. Concave up for $x > -2$. Inflection point at $x = -2$.

§4.3, #25. (a) f has a local maximum at $x = 2$ by the second derivative test.

(b) Since both f' and f'' vanish at $x = 6$, we cannot obtain any information about whether f has a local max, min, or neither at $x = 6$.

§4.3, #34.

$$f(x) = 2 + 3x - x^3 \quad f'(x) = 3 - 3x^2 = 3(1-x)(1+x) \quad f''(x) = -6x$$

(a) Increasing for $-1 < x < 1$. Decreasing for $x < -1$ and $x > 1$.

(b) There is a local minimum at $x = -1$ and a local maximum at $x = 1$. This follows from the first derivative test, or use the second derivative test and note that $f(-1) = 6 > 0$ and $f(1) = -6 < 0$.

(c) Concave up for $x < 0$ and concave down for $x > 0$.

(d) Inflection point for $x = 0$.

§4.3, #45.

$$f(x) = \frac{x^2}{x^2-1} \qquad f'(x) = \frac{-2x}{(x^2-1)^2} \qquad f''(x) = \frac{6x^2+2}{(x^2-1)^3}$$

(a) There are vertical asymptotes at $x = 1$ and $x = -1$ and horizontal asymptotes in both directions along $y = 1$. More precisely,

$$\lim_{x\to 1^+} f(x) = \infty \qquad \lim_{x\to 1^-} f(x) = -\infty$$

$$\lim_{x\to -1^+} f(x) = -\infty \qquad \lim_{x\to -1^-} f(x) = \infty$$

$$\lim_{x\to \infty} f(x) = 1 \qquad \lim_{x\to -\infty} f(x) = 1$$

(b) Increasing for $x < 0$. Decreasing for $x > 0$.

(c) Only critical value is $x = 0$. Since $f''(0) = -2$, it is a local max.

(d) Concave up for $x < -1$ and $x > 1$. Concave down for $-1 < x < 1$. There are no inflection points.

§4.3, #64.

$$f(x) = axe^{bx^2} \qquad\qquad f'(x) = a(1 + 2bx)e^{bx^2}$$

We are told that $f(2) = 1$ and that this is a local maximum, so we also want $f'(2) = 0$. So

$$\begin{aligned} 1 &= f(2) = 2ae^{4b}, \\ 0 &= f'(2) = a(1 + 4b)e^{4b}. \end{aligned}$$

We can't have $a = 0$, and e^{4b} is always positive, so the second equation tells us that $b = \frac{1}{4}$. Substituting that value into the first equation gives $1 = 2ae$, so we find that $a = \frac{1}{2e}$.